

THE LABOR WEDGE AS A MATCHING FRICTION

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ABSTRACT. We use a search and matching model to decompose the labor wedge into three classes of labor market frictions and evaluate their role for the labor wedge and unemployment. We find that there is an asymmetric effect of labor market frictions on the labor wedge and unemployment. While the wedge is mostly explained by changes in matching efficiency, unemployment is accounted for by the combination of frictions to matching efficiency, job destruction and bargaining. If the labor wedge is motivated by search and matching frictions, then it is relevant for explaining unemployment mainly through changes in matching efficiency.

JEL: E20, E32, J22, J63, J64.

Keywords: Labor Wedge, Business Cycles, Search and Matching

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For the last 25 years, macro and labor economists have pointed to large cyclical variations in the relationship between the marginal rate of substitution between leisure and consumption and the marginal product of labor as an important feature of business cycles. In their business cycle accounting framework, Chari, Kehoe, and McGrattan (2007) (CKM) label this relationship a "labor wedge" and argue that it accounts for 60% of output fluctuations.

Apart from the fact that the labor wedge accounts for 60% of output fluctuations, some recent papers (see Shimer (2009)) have pointed out that understanding the labor wedge would give insight into the nature of unemployment. In this paper we ask if labor market frictions that are considered important for explaining unemployment in a search and matching framework are also important for explaining the labor wedge in a general equilibrium model with search and matching frictions.

We consider three labor market frictions which are known to be important in explaining unemployment fluctuations: variations in the job destruction margin as suggested by Mortensen and Pissarides (1994), variations in the bargaining power of workers and firms as in Hall (2005c), and variations in the efficiency of the matching process. Specifically, we look at the labor wedge through the lens of a search and matching model and decompose the wedge into exogenous separation, bargaining, and matching shocks.¹ We use the business cycle accounting methodology to evaluate their quantitative importance.

Our accounting exercise leads us to a striking conclusion: variations in the labor wedge are largely attributed to the matching shock. Other commonly used frictions, such as endogenous variations in job destruction and wage stickiness, play only a minor role in determining the labor wedge.

This implies that the forces that drive unemployment are quite different from those that drive the labor wedge. In particular, imperfections in the job destruction and bargaining processes commonly considered in the search literature are not very helpful

¹Like wedges in CKM, the three additional shocks in our model represent labor market distortions rather than primitive sources of shocks.

in explaining the labor wedge but they are important for understanding unemployment. Likewise, the matching friction explains most of the labor wedge but it is not nearly as important for unemployment. As a result, if the labor wedge is motivated by search and matching frictions, then it is relevant for explaining unemployment mainly through changes in matching efficiency.

Our modeling approach augments the representative agent business cycle model with a search and matching friction in the spirit of Merz (1995) and Andolfatto (1996). The standard assumption that labor is traded in a spot market is replaced by a search friction which puts an additional constraint on how much labor can be employed. Differently from their approach, our model endogenously determines the level of unemployment, the number of vacancies and the labor force participation rate.

To model the frictions mentioned above we introduce three shocks which jointly determine the labor wedge in the model: the *separation shock*, the *matching shock* and the *bargaining shock*. The separation shock represents the proportion of employed workers that get separated from their jobs every period. The matching shock represents the efficiency of the matching technology. The bargaining shock represents the proportions in which the lifetime surplus of a newly formed match is split between the worker and the firm and thus pins down wages.²

To evaluate the relative importance of each shock we use the business cycle accounting methodology employed in Cole and Ohanian (2002) and CKM. For identification purposes, in addition to three labor market shocks, our model includes a TFP shock, an investment shock, and a government consumption shock. We use data on real GDP, consumption, investment, hours, unemployment and vacancies to recover the six shocks: TFP, investment, government consumption, separation, matching and bargaining. We use the model as a diagnostic tool and measure the contributions of each shock to each of the six variables by running a counterfactual exercise: we feed the shocks back into the model one at a time and all but one at a time.

²Note that wage rigidities proposed by Hall (2005c) and Shimer (2010) are one particular case of variations in the bargaining power we consider.

As mentioned earlier, from the counterfactual exercise we conclude that there is an asymmetric effect of labor market frictions on the labor wedge and unemployment. While the labor wedge is mostly explained by changes in the matching efficiency, unemployment cannot be successfully explained without the interaction of the matching, separation and bargaining shocks.

Our results indicate that a labor market friction responsible for variations in the labor wedge must be isomorphic to changes in matching efficiency. This broad class of frictions includes variations in per capita costs associated with creating jobs, variations in time and effort devoted to search by unemployed workers, variations in the level of congestion, and variations in the degree of competition between peers characterizing the matching process.

Note that our results may also point toward an alternative interpretation of the labor wedge. The fact that our model attributes most of the variations in the labor wedge to matching efficiency may be a sign of misspecification of the real business cycle model as discussed by Chang and Kim (2007) and Pescatori and Tasci (2011). We consider this an important area for future research.

The paper is organized as follows. Section 2 lays out the theoretical framework and introduces the six shocks, Section 3 describes the methodology we use to estimate the model and recover the shocks, Section 4 explains the results, and Section 5 concludes.

I. THEORETICAL FRAMEWORK

This section lays out the setup of the model. We modify the standard one sector real business cycle model by adding a search technology for moving labor between productive activities and leisure in the spirit of Merz (1995) and Andolfatto (1996). However, it is well known that introducing search frictions is not enough to explain fluctuations in labor market variables. In order to do that we have to allow for a richer environment.

There are three major adjustments to the search and matching framework which have proven useful for explaining labor market fluctuations. First, Fujita and Ramey (2009) show that variations in the rate of job destruction are empirically relevant for explaining the behavior of the unemployment rate. Second, Hall (2005c) has

shown that variations in the bargaining power of workers, generated, for instance, by wage-stickiness, can help explain the volatility of unemployment and vacancies. Alternatively, Hagedorn and Manovskii (2008) have shown that significant changes in the calibration of the model can help improve its fit of the data. Third, the idea that variations in the efficiency of the matching process play a non-negligible role in unemployment fluctuations and explain the movements of the Beveridge curve is well known at least since Blanchard and Diamond (1989). We allow for all of these three mechanisms by introducing shocks to the separation rate, the bargaining power of workers and to the efficiency of the matching process. We choose to incorporate all three shocks in order to let the data determine their relative importance.

I.1. Model. We assume that the economy is populated by a continuum of families. Each family operates a backyard technology and completely insures its members against variations in their labor incomes. Members of a family cannot work in their own backyard, but can be employed in two market activities: head-hunting which is competitive and a productive activity where the wage is set according to a specific wage-setting rule.

The economy faces six exogenous shocks. A total factor productivity (TFP) shock A_t , an investment-specific technology shock T_t , a government expenditure shock G_t , a shock to the separation rate of employment δ_{Lt} , a shock to matching efficiency B_t , and a shock to the bargaining power of workers ϕ_t . This last shock ϕ_t represents the fraction of the lifetime surplus of the match that goes to the worker, hence, as we will show later on, ϕ_t determines the wage w_t in the productive sector.

At the beginning of period t , values of shocks A_t , T_t , δ_{Lt} , G_t , B_t , ϕ_t , capital K_t , labor supply L_{t-1}^s and demand L_{t-1}^d , and the job-finding and vacancy-filling rates are given. The head of each family decides how many members V_t^s to send to look for a head-hunting job and how many members of other families V_t^d to hire in the head-hunting market to search for unemployed workers to fill in positions in the backyard firm. Each head-hunter represents one vacancy and is paid a wage q_t .

The head of the family also decides how many members L_t^s to send to work in productive activities and how many members of other families L_t^d to employ to operate

her own backyard technology. Finally, the head of the family assigns U_t members to apply for jobs in other families' backyards, allocates resources to consumption C_t of its members and invests into capital K_{t+1} next period.

We introduce head-hunters into the model in order to measure costs of searching for a worker and costs of searching for a job in the same units of disutility of labor. We adopt a specification similar to Farmer and Hollenhorst (2006) where costs of search are in units of labor rather than consumption because it makes the model more tractable. We distinguish between labor demand and supply in both markets in order to derive shadow prices of both types of employment and compute the value of a match.

Each family head maximizes the expected lifetime utility of its members (1), subject to a budget constraint (2) and labor supply and demand accumulation constraints (3) and (4):

$$\max_{\{C_t, L_t^s, L_t^d, V_t^s, V_t^d, U_t, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t^s, V_t^s, U_t), \quad (1)$$

$$C_t + \frac{K_{t+1} - (1 - \delta_K)K_t}{T_t} + G_t \leq A_t F(K_t, L_t^d) + w_t(L_t^s - L_t^d) + q_t(V_t^s - V_t^d), \quad (2)$$

$$L_t^s = (1 - \delta_{L_t})L_{t-1}^s + U_t \frac{\bar{M}_t}{U_t}, \quad (3)$$

$$L_t^d = (1 - \delta_{L_t})L_{t-1}^d + V_t^d \frac{\bar{M}_t}{V_t}, \quad (4)$$

where \bar{M}_t is the total number of matches formed in the economy in period t . In equation (3), labor supply in period t depends on last period's labor supply minus the number of workers that got separated from their job plus the new formed matches. The separation rate δ_{L_t} denotes the exogenously given rate at which workers are separated from their jobs and captures the various frictions leading to variations in job destruction over the cycle. The term $\frac{\bar{M}_t}{U_t}$ stands for the job finding rate and represents the increase in employment when there is one more individual searching for a job (U_t increases by one unit). In equation (4), labor demand accumulates in the same way as labor supply with the difference that the term $V_t^d \frac{\bar{M}_t}{V_t}$ is the vacancy filling rate times the number of head-hunters demanded and means that for every new

individual that works as a head-hunter V_t^d , the stock of employed workers increases by $\frac{\bar{M}_t}{\bar{V}_t}$.

The markets for labor and head-hunting clear when $L_t^s = L_t^d = L_t$ and $V_t^s = V_t^d = V_t$. The law of motion of aggregate employment satisfies

$$L_t = (1 - \delta_{L_t})L_{t-1} + M_t, \quad (5)$$

and in equilibrium $\bar{U}_t = U_t$, $\bar{V}_t = V_t$ and

$$\bar{M}_t = M_t = B_t M(U_t, V_t). \quad (6)$$

In equation (6) B_t represents the efficiency of the matching technology, determining the number of matches formed for each combination of the numbers of workers and head-hunters seeking a match. The resource constraint and production function are given by

$$C_t + \frac{1}{T_t} (K_{t+1} - (1 - \delta_K)K_t) + G_t = Y_t, \quad (7)$$

$$Y_t = A_t F(K_t, L_t^d). \quad (8)$$

We derive the optimality conditions of the model:

$$\frac{1}{T_t} = \beta E_t \frac{U'_{C_{t+1}}}{U'_{C_t}} \left(A_{t+1} F'_{K_{t+1}}(K_t, L_t) + \frac{1}{T_{t+1}} (1 - \delta_K) \right), \quad (9)$$

$$w_t + \frac{U'_{L_t}}{U'_{C_t}} = \mu_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \mu_{t+1} (1 - \delta_{L_{t+1}}) \right), \quad (10)$$

$$A_t F'_{L_t}(K_t, L_t) - w_t = \eta_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \eta_{t+1} (1 - \delta_{L_{t+1}}) \right), \quad (11)$$

$$\frac{U'_{V_t}}{U'_{C_t}} + q_t = 0, \quad (12)$$

$$\eta_t \frac{M_t}{V_t} - q_t = 0, \quad (13)$$

$$-\frac{U'_{U_t}}{U'_{C_t}} = \mu_t \frac{M_t}{U_t}. \quad (14)$$

In the equations above μ_t is the Lagrange multiplier associated with the labor supply accumulation constraint and η_t is the Lagrange multiplier associated with the labor demand accumulation constraint, both measured in units of marginal utility of consumption. Since T_t , A_t , δ_{L_t} , G_t and B_t are exogenous, we have a system of ten equations and eleven variables, $\{K_{t+1}, L_t, C_t, M_t, Y_t, V_t, U_t, \mu_t, \eta_t, w_t, q_t\}$. The model is missing an equilibrium condition because equations (10) and (11) determine two different ways of moving labor between leisure and employment in productive activities, and there is only one price w_t . Therefore, we introduce a bargaining shock to close the model.

I.2. Introducing Bargaining Shocks. We first need to construct the lifetime surplus of a match in order to introduce a bargaining shock that splits this surplus between the worker and the firm. The surplus of a match is defined by the sum of the Lagrange multipliers associated with the labor accumulation constraints. Equations (10) and (11) can be iterated forward to solve for these multipliers:

$$\eta_t = A_t F'_{L_t}(K_t, L_t) - w_t + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left(A_s F'_{L_s}(K_s, L_s) - w_s \right) \prod_{k=t+1}^s (1 - \delta_{Lk}), \quad (15)$$

$$\mu_t = w_t + \frac{U'_{L_t}}{U'_{C_t}} + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left(\frac{U'_{L_s}}{w_s + U'_{C_s}} \right) \prod_{k=t+1}^s (1 - \delta_{Lk}). \quad (16)$$

The disutility of work, $-U'_{L_t}/U'_{C_t}$, equals the value of searching for another job and represents the outside option of a worker. Note that the Lagrange multiplier in the labor demand (supply) accumulation equation is the expected sum of instantaneous marginal values of the match for the representative firm (worker), discounted and adjusted for the probability of the match being dissolved in any given period. Hence, the sum $\Gamma_t = \mu_t + \eta_t$ of the two Lagrange multipliers can be interpreted as the lifetime surplus of the match—an expected sum of instantaneous marginal values of the match, discounted and adjusted for the probability of the match being dissolved in any given period:

$$\Gamma_t = A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}} + E_t \sum_{s=t+1}^{\infty} \beta^{s-t} \left(A_s F'_{L_s}(K_s, L_s) + \frac{U'_{L_s}}{U'_{C_s}} \right) \prod_{k=t+1}^s (1 - \delta_{Lk}). \quad (17)$$

A standard way to close search and matching models is to assume that the worker and the firm use Nash bargaining over the wage and split the surplus in constant proportions. However, Shimer (2005a) and Hall (2005a) argue that for a Mortensen-Pissarides-type model to fit the data one needs variations in the bargaining power. In order to incorporate such a friction we close the model by assuming that the surplus Γ_t is split between the worker and the firm according to a time-varying rule.³

We define a variable ϕ_t to represent the fraction of the lifetime surplus of the match going to the worker and $(1 - \phi_t)$ the fraction of the lifetime surplus of the match going to the firm. We assume that $\eta_t = (1 - \phi_t)\Gamma_t$ and $\mu_t = \phi_t\Gamma_t$. Hence, we refer to ϕ_t as the bargaining power of the worker as well as the bargaining shock. Notice that allocations are economically inefficient whenever ϕ_t is different from the elasticity of the matching function with respect to the number of unemployed.⁴ This way of splitting the surplus implies a wage-setting rule which is general enough to capture various mechanisms of wage adjustment, including wage rigidity proposed by Hall (2005a) (see Appendix A.1.2).

A competitive equilibrium of the model economy is a solution to equations (5)-(14), together with the bargaining condition, where $\{K_{t+1}, C_t, L_t, V_t, U_t, \Gamma_t, w_t, q_t, \Psi_t, Y_t, M_t\}$ are endogenous variables and $\{A_t, T_t, \delta_{L_t}, G_t, B_t, \phi_t\}$ are the exogenous shocks of the model. The exogenous variables behave according to a stochastic process to be defined later.

I.3. Identifying assumptions. Most of the functional forms we use are standard in the literature. We assume that the production and matching functions are Cobb-Douglas with constant returns to scale:

³For a more general description of how we construct the bargaining shock, see Appendix A.

⁴See Hosios (1990) for a detailed discussion.

$$F(K, L) = K^\alpha L^{1-\alpha}. \quad (18)$$

$$M(U, V) = U^\theta V^{1-\theta}. \quad (19)$$

We postulate a utility function consistent with a balanced growth path, where fractions of time spent head-hunting and searching for a job enter symmetrically with the time spent on the production activity:

$$U(C, L, U, V) = \log C - \chi \frac{(L + V + U)^{1+\gamma}}{1 + \gamma}. \quad (20)$$

This functional form implies that workers get the same disutility from working in productive activities as when searching for a job or head-hunting. We assume that although individuals spend only a few hours per week searching for a job, they also spend time in other activities that generate disutility: in expanding their network by making phone calls, getting technical training, continuing their education, helping their relatives or working in home production.

An implication of this assumption is that the costs of searching for a job from the point of view of an unemployed worker and of a firm with a vacant position are equalized. While micro data sheds little light on how to discipline these costs, this assumption delivers a clear-cut interpretation of the shocks. In the decomposition we perform this assumption distinguishes the variations in labor market variables attributed to the bargaining shock from those attributed to the matching shock. It implies that any variations in the search costs on the worker and firm sides show up as variations in the matching efficiency shock, while variations in the wage-setting practices are reflected in the bargaining shock.

Given these functional forms and stochastic processes for the shocks (to be defined later) the shocks are uniquely identified. Appendix A explains step by step how, given data on output, consumption, investment, hours, unemployment and vacancies, one can recover the shocks.

I.4. The Labor Wedge. In this subsection we show that the labor wedge can be decomposed into three labor market shocks which jointly determine its behavior. The wedge itself is defined as the ratio of the marginal product of labor and the marginal rate of substitution between consumption and leisure

$$1 + \tau_t^L \equiv \frac{MP_t}{MRS_t} \equiv \frac{A_t F'_{L_t}(K_t, L_t)}{-U'_{L_t}/U'_{C_t}}. \quad (21)$$

Combining equations (12)-(14) and substituting in the functional forms, we get:

$$\frac{1 - \phi_t}{\phi_t} = \frac{V_t}{U_t}, \quad (22)$$

$$MRS_t = B_t \phi_t^\theta (1 - \phi_t)^{1-\theta} \Gamma_t. \quad (23)$$

Equation (22) shows that the bargaining shock directly pins down market tightness. This is a standard implication of models with rigid wages. It is also a common result in general equilibrium models with search and matching frictions. For instance, Blanchard and Gali (2010) show that when fluctuations in productivity affect the outside option of the worker, then the way the surplus is split determines market tightness.

Equation (23) shows the effects of the matching and bargaining shocks on the marginal rate of substitution. Substituting these into equation (17), moving everything except the marginal product to one side, and rearranging, we get:

$$\tau_t^L = \frac{1}{B_t \phi_t^\theta (1 - \phi_t)^{1-\theta}} - \beta E_t \frac{U'_{C_{t+1}}}{U'_{C_t}} \frac{MRS_{t+1}}{MRS_t} \frac{1 - \delta_{Lt+1}}{B_{t+1} \phi_{t+1}^\theta (1 - \phi_{t+1})^{1-\theta}}. \quad (24)$$

Equation (24) defines the labor wedge and shows how it depends on the separation, bargaining and matching shocks. First, we can see that separation shocks only affect the labor wedge through their impact on the value of a match between the worker and the firm. Because separation shocks affect the way future payoffs are discounted, only expectations on future separation shocks rather than the shocks themselves affect the labor wedge.

Second, we can see from equation (23) that given the constant returns to scale in the matching function, the positive effect that bargaining has on unemployment is offset by the negative effect it has on vacancies. Note that when the Hosios condition holds ($\phi_t = \theta$), then bargaining shocks do not affect the labor wedge. Finally, lower matching efficiency implies a bigger wedge.

II. METHODOLOGY

Our methodology follows that of CKM. We use data together with the optimality conditions of the model to pin down the shocks. We solve equations of the model using data on output, consumption, investment, hours, unemployment and vacancies to compute the six shocks (see Appendix A.1.3). If we fit the shocks back into our model we recover the original data.

Unlike in CKM, expectations of future values of a large number of variables enter into many of the equations of the model. Given this, we use a maximum likelihood estimation procedure and apply the Kalman filter to a linearized version of the model to compute the values of the shocks. We use Bayesian estimation to simultaneously recover the processes for the shocks and some of the parameters of the model.

Just as in CKM, to evaluate the effect of each of the shocks we conduct a counterfactual experiment where we simulate the economy with that shock fixed at its initial value. Each experiment isolates the direct effect of the shock, but retains its forecasting effect on the other shocks. This procedure ensures that the expectations of the shocks are identical to those in a model where all the shocks are present at the same time.

II.1. Processes for the shocks. In the data real output, consumption and investment are nonstationary even with respect to a log-linear trend.⁵ To make the data comparable to the model, we follow the approach presented in Fernandez-Villaverde and Rubio-Ramirez (2007) and assume random walks for the two processes that are commonly thought to be extremely persistent: the TFP and investment shocks A_t

⁵Cogley and Nason (1995) and Canova (1998) show that the use of the Hodrick-Prescott filter introduces significant biases into the data by amplifying business-cycle frequencies.

and T_t . Thus, the growth rates of TFP and investment shocks are assumed to follow first-order autoregressive processes.⁶ We denote a_{ss} the mean growth rate of TFP and τ_{ss} the mean growth rate of the investment-specific technology.

From the optimality conditions of the model we can see that all variables except capital grow at a factor $(a_{ss}\tau_{ss})^{\frac{1}{1-\alpha}}$. Then, if we take the first differences of the TFP and investment shocks by defining $a_t = \frac{A_t}{A_{t-1}} = a_{ss} \exp(\sigma_A \varepsilon_{At})$ and $\tau_t = \frac{T_t}{T_{t-1}} = \tau_{ss} \exp(\sigma_T \varepsilon_{Tt})$, we can derive an aggregate trend $Z_t^{1-\alpha} = A_t T_t^\alpha$, which is common to all the variables except capital. Hence, we define detrended variables of the form $x_t = \frac{X_t}{Z_t}$. Capital grows at a factor $(a_{ss}\tau_{ss})^{\frac{1}{1-\alpha}}$, so it is detrended as follows: $k_{t+1} = \frac{K_{t+1}}{Z_{t+1}}$. Appendix B shows the resulting detrended equilibrium conditions of the model.

We assume that the rest of the shocks follow first-order autoregressive processes around their steady-state values. In order to allow for correlations between both innovations and levels of the shocks in our model, we estimate a vector autoregression (VAR) for the six shock processes. This simple procedure, as in CKM, allows us to estimate the interaction between the levels of shocks and the correlation structure of innovations to these shocks as follows. Let X_t denote the six-by-one vector of recovered shocks. We employ the following auto-regressive specification:

$$X_t = PX_{t-1} + Qu_t, \quad u_t \sim N(0, \Sigma). \quad (25)$$

In the first step, we assume a diagonal VAR specification to pin down expectations. We estimate the diagonal elements jointly with the parameters of the model. In the second step, we use shocks recovered from the first step to estimate the off-diagonal parameters of the VAR specification. In the third step, we incorporate the estimated transition matrix, P , and the correlation structure of innovations, Q , into our model and re-estimate it holding all off-diagonal coefficients in P and Q fixed. The outlined procedure allows us to capture the contemporaneous interaction of the shock processes and their effects on expectation formation in the model. We use this

⁶Using an HP-filter does not change any of our main results.

estimated specification to run the counter-factuals and measure the contributions of different shocks to different variables.⁷

II.2. Data. We use six variables in our estimation procedure: 1) real per capita GDP, 2) real per capita nondurable consumption expenditures, 3) real per capita gross private domestic investment (including durable consumption), 4) an index of aggregate weekly per capita hours worked in private industries, 5) the unemployment rate, and 6) the Conference Board help-wanted advertising index (merged with JOLTS job openings data after 2001) as a proxy for vacancies.

All data are seasonally adjusted. Monthly data is averaged to make it quarterly. We divide by population to obtain per capita values. This corresponds to modeling the economy using a representative household/firm. We take logs of GDP, consumption and investment and then take the first difference. We remove long-run secular trends from hours, unemployment and vacancies, which are a result of demographic and other factors unrelated to business cycles.⁸ We normalize the resulting detrended indices of hours and vacancies to one on average. All data we use is for the period 1951:I-2011:IV.

To be able to estimate the model we need to add six measurement equations corresponding to the six variables that we observe. Since the data for real output, consumption and investment are modeled as nonstationary, we take the first differences of the data to make it comparable to the model. In addition, the definition of output in our model includes time spent head-hunting. In the real economy firms are paying head-hunters a wage and it is measured as part of GDP. To account for this, we derive the price of time spent head-hunting, multiply it by the amount of time spent in this activity and include the product in our definition of GDP.

Hours in our model correspond to the total time spent on the productive activity and head-hunting. This index corresponds closely to total employment $L_t + V_t$, since

⁷Although the innovations u_t cannot be treated as structural when a VAR is used, there is always a unique combination of these innovations which would give any particular path of shocks used for the counter-factual exercise. This is the route that we take when measuring the contributions of each shock.

⁸We use an hp-filter with a smoothing parameter 100000 (we follow Shimer (2005a)).

most of the cyclical variation in hours is on the extensive margin (see Gertler, Sala, and Trigari (2008) and Hall (2005a)).⁹ Due to the above correspondence between hours and employment, the time spent by the representative agent searching for a job as a fraction of the total time spent in the labor market $\frac{U_t}{L_t+V_t+U_t}$ corresponds to the number of people searching for a job as a fraction of people participating in the labor market—the unemployment rate. Changes in the help-wanted advertising index proxy changes in the number of vacancies V_t posted by firms.

II.3. Calibration and Estimation. Our model has 9 structural parameters and 14 parameters that characterize the shocks. There are four parameters standard to the business-cycle literature that we calibrate. We set the share of capital in the Cobb-Douglas production function α to 0.34, the discount factor β to 0.99, the depreciation rate δ_K to 2.5% per quarter. We set the value of the Frisch elasticity of labor supply to 0.5 following CKM. We set the steady-state value of the government shock to 22% of GDP, the average value in the data. We also set the elasticity of matches to unemployment θ to 0.7, the value used by Shimer (2005a); this falls within the range of values plausible from a microeconomic perspective reported by Blanchard and Diamond (1989). We calibrate this parameter because it is not well-identified, i.e. the prior is not different from the posterior if the parameter is estimated jointly with other parameters.

We calibrate the steady-state separation rate to be 4%. This is lower than Shimer’s (2005a) quarterly estimate of the separation probability for employed workers. This difference comes from the fact that our separation rate corresponds to the average fraction of jobs permanently destroyed every quarter. In addition to the permanent destruction, an estimate of the separation rate would include a component capturing short-term turnover between employment and unemployment and a large job-to-job transition component. Assuming (following Shimer) that the average job finding rate is 40% per month and that the separation rate is 3% per month, the effective number

⁹We have estimated the model using data on total employment instead of total hours. Most of our results remain unchanged. We prefer using hours so that we can directly compare our results to CKM.

TABLE 1. Calibrated parameters

α	β	δ_K	g_{ss}	θ	a_{ss}	τ_{ss}	δ_{Lss}	γ
0.34	0.99	0.025	0.22	0.7	1.0016	1.0012	0.04	0.5

TABLE 2. Prior and posterior distributions of structural parameters

Parameter	Prior			Posterior		
	Distribution	Mean	S.D.	Mean	[5%	95%]
ω_{ss}	Gamma	0.35	0.150	0.62	[0.57,	0.68]
ϕ_{ss}	Beta	0.50	0.200	0.82	[0.79,	0.84]

of people becoming and staying unemployed until next quarter should be around 2-4%, which is consistent with our estimate.

From the average growth rates of investment, consumption and output, we infer the means of innovations to TFP and investment shocks. We calibrate them to be 0.16 percent and 0.12 percent per quarter, respectively. Table 1 summarizes the calibrated parameters.

We estimate the model using Bayesian methods (see An and Schorfheide (2007)). Linearized equations of the model combined with the linearized measurement equations form a state-space representation of the model. We apply the Kalman filter to compute the likelihood of the data given the model and to obtain the paths of the shocks. We combine the likelihood function $L(Y^{Data}|p)$, where p is the parameter vector, with a set of priors $\pi_0(p)$ to obtain the posterior distribution of the parameters $\pi(p|Y^{Data}) = L(Y^{Data}|p)\pi_0(p)$. We use the Random-Walk Metropolis-Hastings implementation of the MCMC algorithm to compute the posterior distribution.

Table 2 reports the prior and posterior distributions of each structural parameter. The parameter ω_{ss} represents the steady-state job finding rate. Our model implies a 62% average quarterly job-finding rate which is comparable to Shimer's estimates and is consistent with the average duration of unemployment in the U.S. economy.

We estimate the steady-state bargaining power ϕ_{ss} to be 0.82, which is relatively high compared to the value of 0.5 common in the literature (see Mortensen and Nagypal (2007) and Hall (2005a)). The estimates of the two parameters ω_{ss} and

TABLE 3. Prior and posterior distributions of shock parameters

Parameter	Prior			Posterior		
	Distribution	Mean	S.D.	Mean	[5%,	95%]
ρ_A	Beta	0.00	0.05	-0.06	[-0.22,	0.07]
ρ_T	Beta	0.00	0.05	0.03	[-0.26,	0.18]
ρ_S	Beta	0.50	0.20	0.66	[0.53,	0.74]
ρ_M	Beta	0.80	0.10	0.88	[0.85,	0.92]
ρ_B	Beta	0.80	0.10	0.92	[0.91,	0.94]
ρ_G	Beta	0.80	0.10	0.93	[0.91,	0.97]
σ_A	IGamma	0.02	0.010	0.0093	[0.0068,	0.0114]
σ_T	IGamma	0.02	0.010	0.0083	[0.0075,	0.0094]
σ_S	IGamma	0.25	0.100	0.137	[0.092,	0.163]
σ_M	IGamma	0.10	0.050	0.048	[0.031,	0.059]
σ_B	IGamma	0.10	0.050	0.029	[0.025,	0.034]
σ_G	IGamma	0.08	0.040	0.024	[0.020,	0.027]

ϕ_{ss} jointly imply that the average reservation utility is approximately 90% of the worker's marginal product. This moves in the direction of Hagedorn and Manovskii's (2008) calibration of the value of non-market activity (0.95) and is higher than the calibration of Hall (0.4). Our estimate of the parameter ω_{ss} also pins down the ratio of time spent head-hunting to time spent in the production activity which turns out to be 4%. Taking into account the proximity of the shadow prices of different allocations of time, this mimics closely Hagedorn and Manovsky's estimate of the cost of vacancies being 3-4.5% of the quarterly wage. However, unlike their model, a lot of the variation in the bargaining set comes from variations in the value of non-market activity rather than the marginal product.

Table 3 reports the prior and posterior distributions of the persistence and variance parameters of the shocks. The separation rate is the least persistent with a quarterly autoregressive parameter equal to 0.66. The bargaining, matching and government shocks are more persistent, but still significantly less persistent than a random walk.

TABLE 4. Parameters of the Vector AR(1) Stochastic Process

Coefficient Matrix P on Lagged States						
	Separation	Bargaining	Matching	TFP	Invest-t	Gov-t
Separation	0.613	0.804	1.260	-10.41	-3.072	0
Bargaining	0.105	0.920	-0.026	-2.650	0.338	0
Matching	-0.063	0.073	0.887	-2.396	-1.682	0
TFP	-0.001	0.019	0.011	-0.079	-0.183	0
Invest-t	-0.006	0.020	0.044	0.057	0.091	0
Gov-t	0	0	0	0	0	0.934

Correlation Matrix V of Innovations, where $V = Q'Q$						
	Separation	Bargaining	Matching	TFP	Invest-t	Gov-t
Separation	1	0.51	-0.42	-0.12	-0.24	0
Bargaining	0.51	1	-0.67	-0.38	-0.22	0
Matching	-0.42	-0.67	1	0.71	0.33	0
TFP	-0.12	-0.38	0.71	1	0.29	0
Invest-t	-0.24	-0.22	0.33	0.29	1	0
Gov-t	0	0	0	0	0	1

The persistence of government consumption is 0.93 — close to that inferred directly from the data. See Figures in Appendix C to compare the prior and posterior distributions of the parameters. Finally, Table 4 shows the estimated parameters of the VAR representation of the shock processes. As in CKM, we assume that the process for the government shock is orthogonal to the other shocks. Recall that TFP and investment shocks represent growth rates. As a result, it is natural to expect autoregressive coefficients around zero.

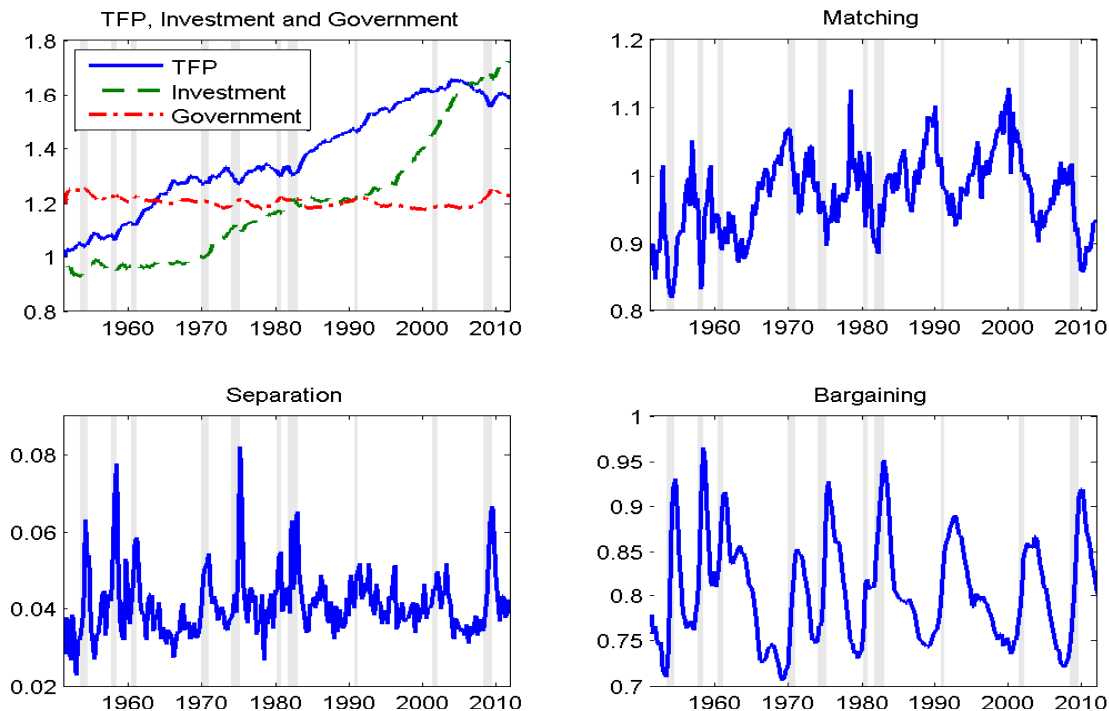
Our model explains by construction 100% of the variation in the data and thus provides the decomposition we need for the business cycle accounting exercise.

III. RESULTS

We divide the discussion of our results into three parts. In the first part, we characterize the behavior of the underlying shocks. We provide a detailed description

of the identification strategy that leads to these results and compare the model-generated series for wages and worker transition rates to the data.

FIGURE 1. The six shocks



The second part constitutes the analytic core of our results. We point out that the labor wedge can be interpreted as the instantaneous welfare gain from a new match. We show that this gain shrinks in good times and expands in recessions, this implies a countercyclical labor wedge as in CKM. We measure the contributions of the three labor market shocks and evaluate their effects on the labor wedge. We show analytically why matching efficiency shocks are the most relevant for explaining the dynamics of the labor wedge.

In the third part we analyze the effects that separation, matching and bargaining shocks have on output and unemployment. The quantitative impacts of the three shocks are in agreement with several well known mechanisms described in the labor literature. First, the spikes in separations account for the sharp increases in unemployment at onsets of recessions as they do in the data. Second, the behavior of the

shock to bargaining power is consistent with the wage rigidity theory of Hall. Third, declines in matching efficiency contribute to jobless recoveries as in the mismatch literature.

Overall, we find that matching efficiency plays only a minor role in unemployment fluctuations, but it is the main driving force behind the labor wedge. This leads us to the conclusion that the driving forces behind variations in unemployment are quite different from those behind the labor wedge.

III.1. Behavior of the Underlying Shocks. Figure 1 describes the behavior of the recovered shocks over the whole sixty-year period. The shaded vertical areas correspond to the official recession periods according to the NBER. Note that TFP and investment shocks are random walks with drifts, while the rest of the processes are stationary.

We find that total factor productivity slows down at the beginning of each recession. The investment-specific technology tends to increase in recessions and has a negligible effect on output and the labor market variables. This supports the main finding of CKM, that the investment wedge plays only a tertiary role in U.S. business cycles. The government shock, as well as the investment shock, only affects consumption and investment. Because we are primarily interested in the behavior of output, hours, unemployment and vacancies, for the rest of the exposition we abstract from the behavior of investment and government shocks. Instead we focus on technological shocks and shocks that constitute the labor wedge.

Figure 1 shows that the separation rate tends to be high at early stages of each recession.¹⁰ The wave of separations typically starts earlier than the recession itself and dies out quickly—within a year after the start of a recession.

Towards the end of recessions matching efficiency tends to decrease, which leads to a decline in match formation and causes the amount of hours worked to fall. While the outside option of the worker tends to decrease in recessions because of a decrease in their marginal disutility of work, a corresponding increase in the bargaining power

¹⁰As a consistency check note that these spikes in the separation rate, in general, coincide with spikes observed in Shimer’s data.

tends to move wages in the opposite direction. Thus, our finding that bargaining power of workers increases significantly during recessions is consistent with a view of wage rigidities as a major source of inefficiency in the labor market.

Let us now take a closer look at the timing of shocks. From Figure 1 it is clear that declines in TFP slightly precede increases in the separation rate. An increase in the separation rate is typically followed by an increase in the bargaining power of workers which precedes or coincides with a decrease in the matching shock (see also cross-correlations in Appendix D). This implies that shocks to the separation rate are important at early stages of recessions, and bargaining and matching shocks come into play later.

Why does our business cycle accounting methodology recover significant variations in the separation rate, in the matching efficiency and in the bargaining power? We believe that this is a general result in models where agents decide on the margin. More precisely, we argue that models where workers and firms equalize benefits and costs of searching for a job and opening a vacancy would predict sizable changes both in shocks and incentives.

When workers choose whether to search for a job (equation (14)), they equate the cost of searching for a job—which is equal to the MRS in our model—with the potential benefits of forming a match times the probability of finding a job. The benefits are equal to the present discounted value of the wages minus the cost of working, which is also equal to the MRS:

$$MRS_t = \phi_t \Gamma_t \frac{M_t}{U_t} = PV(W_t - MRS_t) \frac{M_t}{U_t}. \quad (26)$$

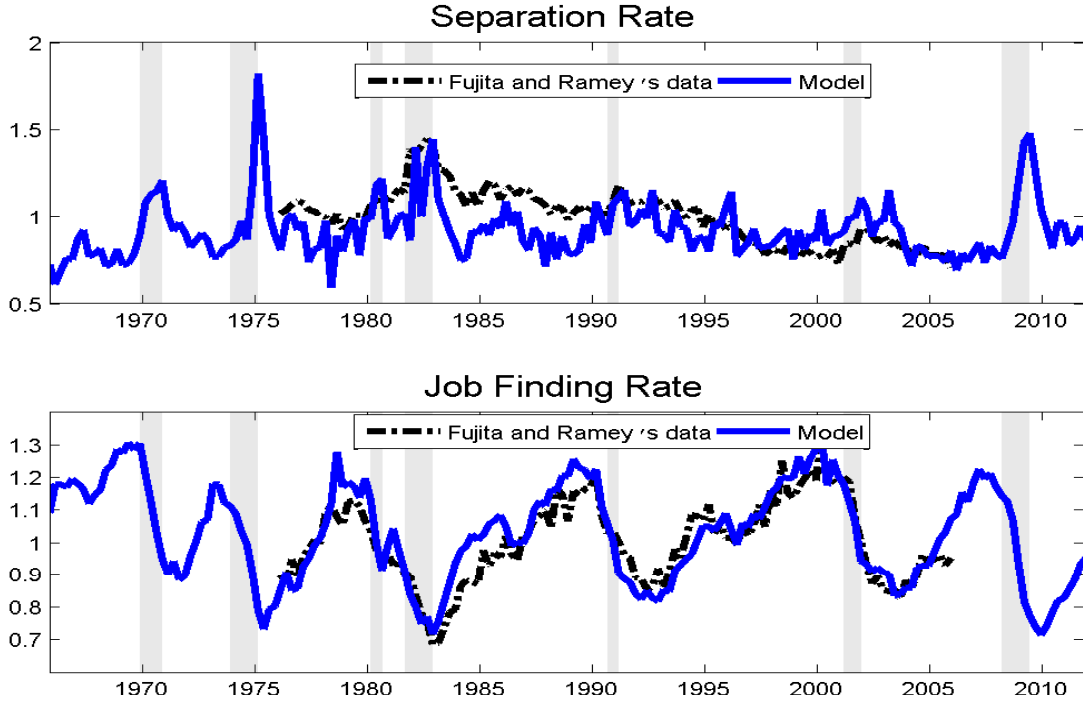
Given that in the data the job finding probability $\frac{M_t}{U_t}$ declines significantly in recessions (documented by Shimer (2005a)) and the wage is relatively smooth, equation (26) implies that the MRS has to fall by a fair amount. In the model, swings in the MRS are due to the elasticity of the utility function of 0.5, common in the RBC literature.

Secondly, notice that when firms choose whether to open a new vacancy, they also equate the competitive salary they pay to a head-hunter with the potential benefits

of forming a match times the probability of finding a worker to fill the vacancy. The benefits are equal to the present discounted value of the marginal product minus the wage that they pay to the worker:

$$MRS_t = (1 - \phi_t) \Gamma_t \frac{M_t}{V_t} = PV (MP_t - W_t) B_t \left(\frac{U_t}{V_t} \right)^\theta. \quad (27)$$

FIGURE 2. Comparison of Worker Transition Rates to the Data



Given that we have already established that the MRS decreases in recessions, and taking into account the fact that in the data unemployment increases, while the number of vacancies falls and both the wage and the marginal product are not very volatile, equation (27) implies that the matching efficiency has to fall in recessions.

Combining equations (26) and (27) one can find that the bargaining power of the workers is directly pinned down by the market tightness:

$$\frac{1 - \phi_t}{\phi_t} = \frac{V_t}{U_t}. \quad (28)$$

Thus, when unemployment increases and there are fewer vacancies, the bargaining power of workers has to increase by a comparable amount. Variations in the separation rate that we estimate are a residual of the labor accumulation equation in the productive sector. They have large spikes at onsets of recessions, which is a common feature of the data on separations.

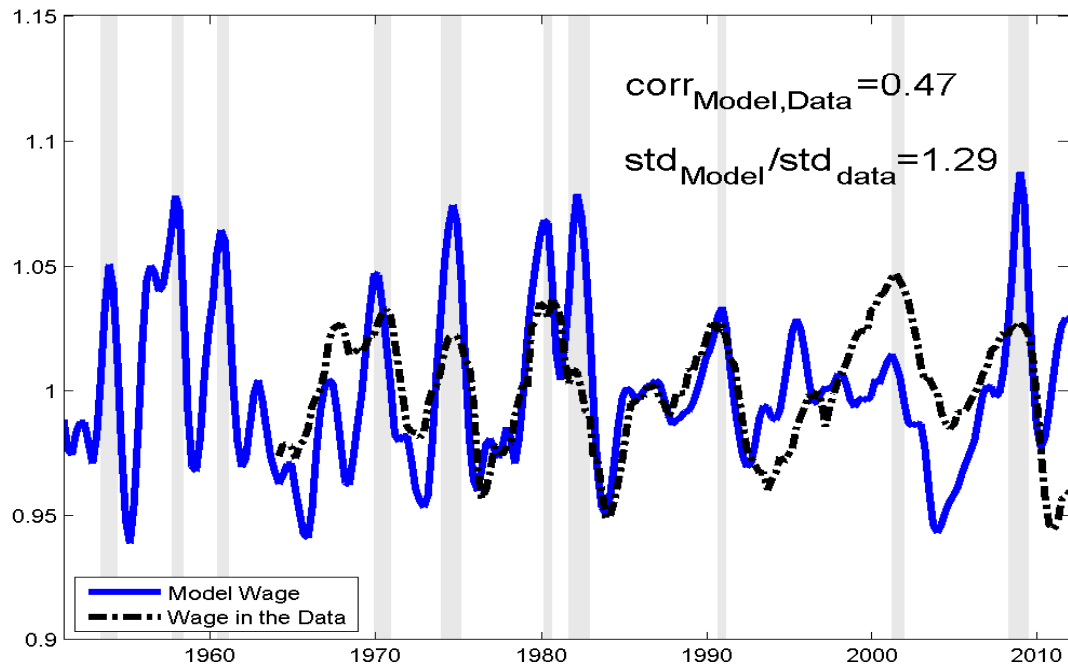
Figure 2 compares the model-generated worker transition rates to the data on separation and job finding rates constructed from the household survey by Fujita and Ramey (2006) for the period from 1976:I to 2005:IV. It shows that the model predicts reasonable fluctuations both in the rate at which employed workers lose jobs and the rate at which unemployed workers find new jobs. The model explains the large swings in the job finding rate and predicts spikes in the separation rate similar to those observed in the data. Figure 2 demonstrates that the recovered shocks to the separation rate and to matching efficiency capture the mechanisms behind labor market movements.

To summarize, for a model where both households and firms decide on the margin how much time to spend searching for each other to match aggregate data, one needs to generate changes in both the marginal rate of substitution between consumption and leisure and the bargaining power of workers. Procyclical reservation values, along with countercyclical bargaining power of workers, help match the volatile behavior of unemployment and vacancies and predict mild fluctuations in wages.

As an additional over-identifying restriction on our model, we use the observation made by Shimer (2005a) that a standard Mortensen-Pissarides-type model, when hit by productivity shocks of plausible magnitude, predicts wages to be much more volatile than in the data, while generating relatively small variations in unemployment and vacancies.

Our model fits the volatility of unemployment and vacancies by construction. Figure 3 depicts the behavior of wages predicted by the model and compares it to the data (adjusted for the stochastic trend). The model predicts wages that are about 30 percent more volatile than in the data, and the correlation between the two is high

FIGURE 3. Wages: model versus data

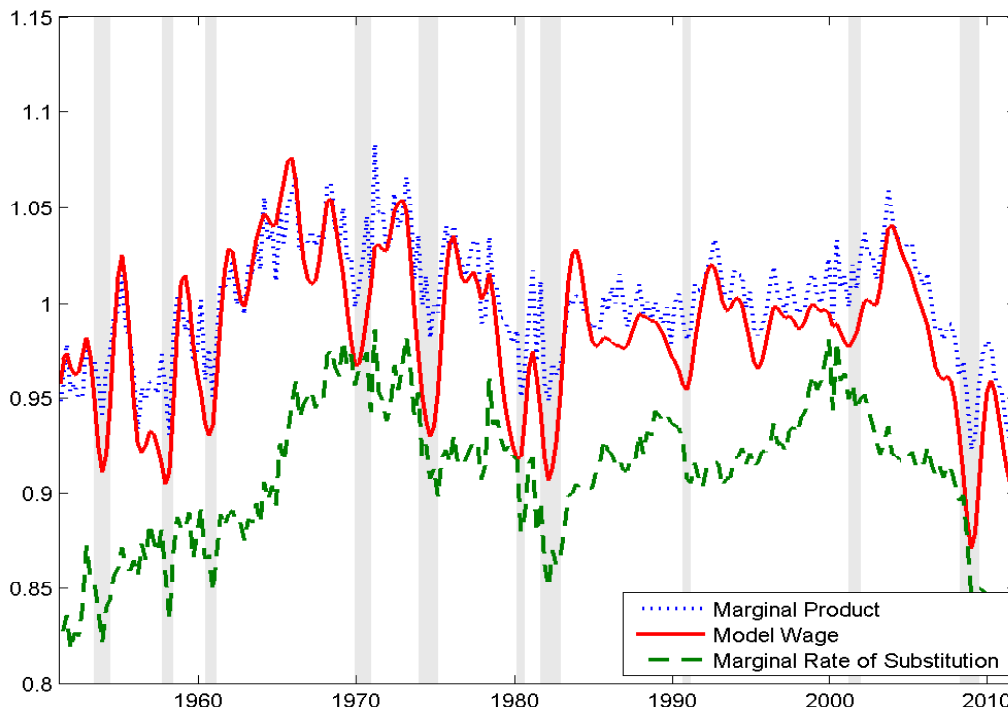


(0.47). Note that the ability of a search model to match volatilities of unemployment and vacancies does not automatically imply matching the behavior of wages, as shown by Lubik (2009). It is important to consider that we are not using data on wages and transition rates in the estimation procedure. The ability of the model to generate all three series so similar to the observed ones is remarkable.¹¹

The predicted wage level splits the instantaneous value of the match between the worker and the firm in the proportion of their bargaining weights, as illustrated by Figure 4. Figure 4 also demonstrates that while the reservation value of workers falls in recessions, wages fall less, thus indicating that the bargaining power of workers increases in recessions. This result supports wage rigidity as a mechanism behind the large changes in the bargaining power of the workers. However, unlike previous

¹¹Though the fit is not perfect, our predicted wage series is much closer to the actual wage data than predictions of existing models, which focus on matching just two moments of the data: the volatility and correlation of wages with labor productivity.

FIGURE 4. Variations in the bargaining set



models of Hall (2005c) and Farmer and Hollenhorst (2006), where increases in the bargaining power in recessions were a result of declines in the marginal product combined with wage rigidity, in our model they are a consequence of declines in the reservation value (MRS) together with wage rigidity.

Thus, allowing for changes in the marginal rate of substitution between consumption and leisure and, consequently, for changes in the reservation value of workers, our model both matches the volatile behavior of unemployment and vacancies and predicts an absence of significant fluctuations in wages, just as in the data. Hence, by allowing for variations in the outside option of workers, our model provides a mechanism which has the potential to solve Shimer's puzzle.

III.2. The Labor Wedge. Following most of the literature, we define the labor wedge as the ratio of the marginal rate of substitution between leisure and consumption (MRS) and the marginal product of labor (MP). Figure 4 depicts the behavior

of these two determinants of the labor wedge. We can see that most of the volatility of the labor wedge comes from variations in the marginal rate of substitution, rather than the marginal product.

In the context of our model, the labor wedge has a new interpretation. The MRS represents the reservation value (outside option) of workers when bargaining over the wage, which implies that the difference between the MP and the MRS represents the instantaneous welfare gain of a new match.¹² It is clear from Figure 4 that the bargaining set narrows in good times and widens in recessions. Thus, in bad times the labor wedge widens, reflecting an increase in the value of new matches and vice versa. In other words, the labor wedge is counter-cyclical, as in the literature.

To measure the contribution of each shock to the labor wedge, we run counterfactual experiments where we fix the levels of the shocks one by one and simulate the model. We obtain paths of the labor wedge which would have taken place if only one distortion was absent. This exercise allows us to compare the actual path to a hypothetical path in a world where one of the imperfections is absent.

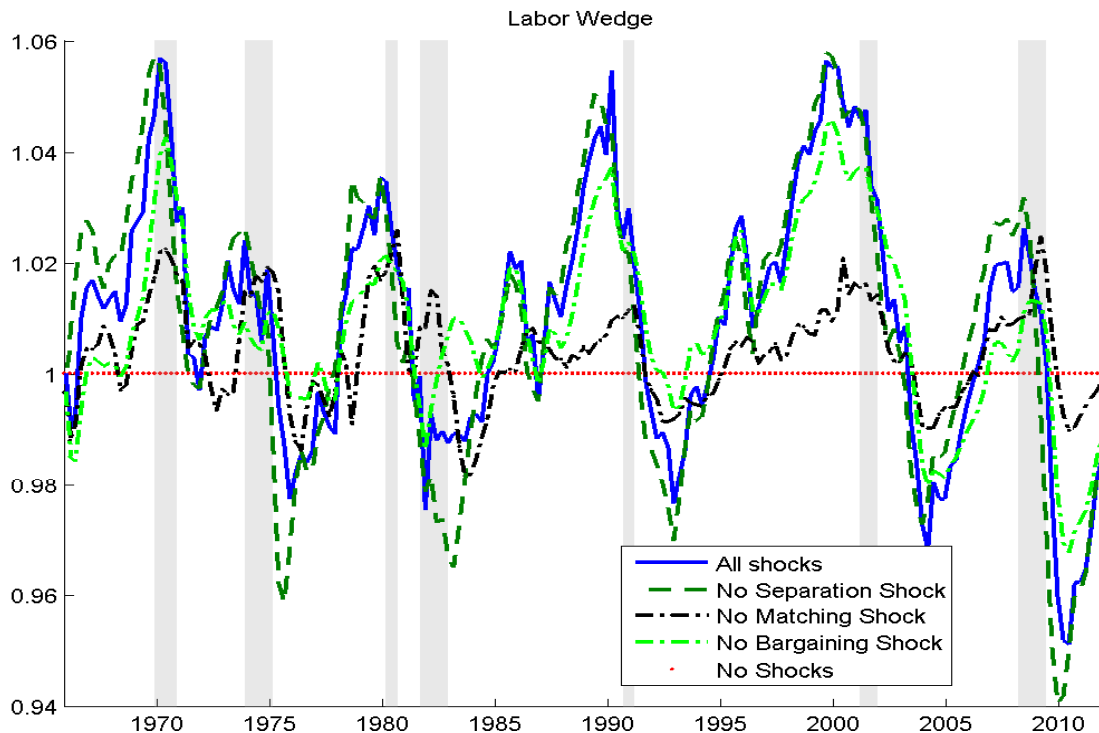
Fixing the levels of each one of the labor market shocks reveals a striking picture. Figure 5 shows that the absence of separation and bargaining shocks has only a mild effect on the behavior of the labor wedge, while the absence of shocks to matching efficiency substantially reduces its volatility. Hence, matching shocks play a dominant role at explaining the labor wedge.

Recall equation (24) which shows how the separation, bargaining and matching shocks jointly determine the labor wedge. Note that when the separation shock is non-persistent, it should not play a significant quantitative role, since only its expectation affects the labor wedge. This is true because agents expect the separation rate next period to be in the neighborhood of the steady-state. However, in general both matching and bargaining shocks can have a substantial effect on the labor wedge.

Changes in the matching shock are always going to matter, while the importance of the bargaining shock depends on the relationship between θ and the steady-state value of ϕ_t . Notice that an increase in the bargaining power of workers always leads

¹²The behavior of the instantaneous gain is very similar to that of the lifetime gain.

FIGURE 5. The decomposition of the labor wedge



to a corresponding decrease in the bargaining power of firms and the total effect of the term $\phi_t^\theta (1 - \phi_t)^{1-\theta}$ depends on θ . Equation (29) states that if ϕ_{ss} is equal to θ then the effects of the bargaining shocks are small:

$$\left. \frac{\partial \phi_t^\theta (1 - \phi_t)^{1-\theta}}{\partial \phi_t} \right|_{\phi_t = \theta} = \left(\frac{\theta}{\phi_t} - \frac{1 - \theta}{1 - \phi_t} \right) \phi_t^\theta (1 - \phi_t)^{1-\theta} \Big|_{\phi_t = \theta} = 0. \quad (29)$$

This implies that even when the Hosios condition does not hold exactly, but holds on average, changes in the bargaining power should not significantly affect the labor wedge. In fact the values of θ and ϕ_{ss} have to be very far apart for the bargaining power to have a substantial effect on the labor wedge. Therefore it is natural to expect matching shocks to play a dominant role in determining the behavior of the wedge.

Our calibration of θ of 0.7 as suggested by Shimer and our estimate of the steady-state value of the bargaining power of 0.82 are not that far apart. Hence, our result

that the bargaining shock has only a limited effect on the labor wedge is not surprising.

III.3. Decomposition of Output and Unemployment. To analyze in detail the effects of each shock on output and unemployment and the timing patterns, we focus on the 2008 recession episode, which is the last recession in our sample. We use this recession to illustrate our results as it is easier to see the results in a more detailed graph than it is to see them in a graph containing the whole period. At the end of the section, we show that the results hold for all recession periods in the sample.

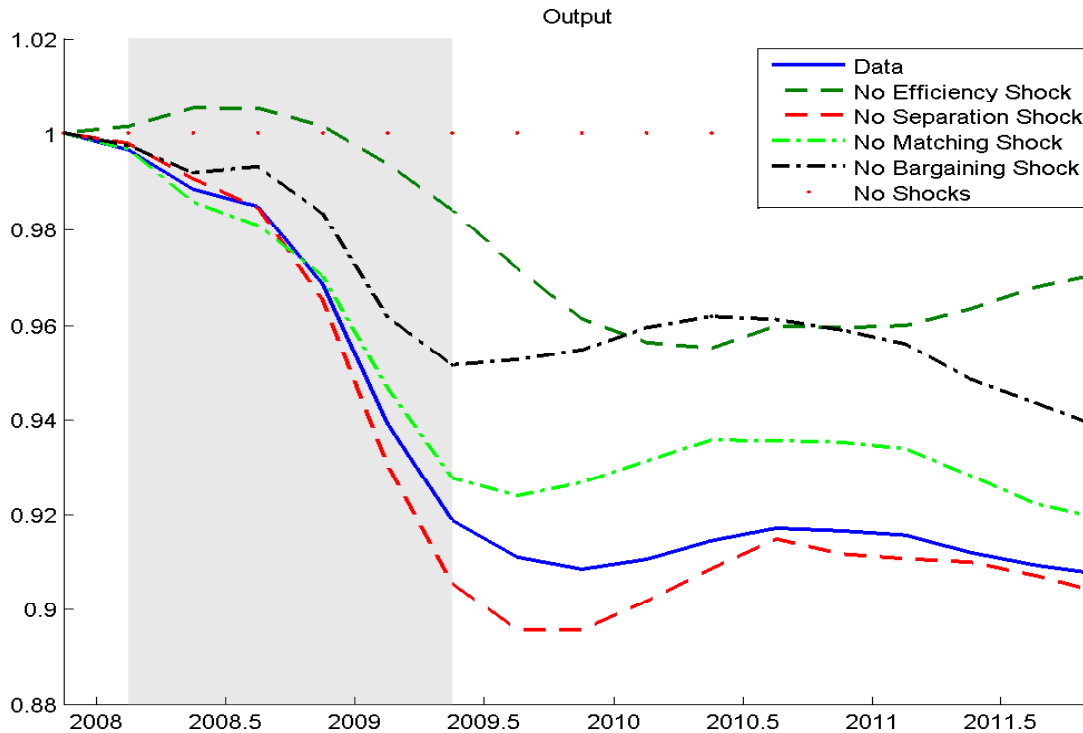
As in the previous subsection, we compare the actual path of GDP with paths it would have taken if we eliminated effects of just one of the shocks. Similarly to the finding that matching shocks play a major role in the behavior of the labor wedge, the impact on output is also relatively clear-cut. Figure 6 illustrates the effects of shocks to TFP, the separation rate, the bargaining power of workers, and the matching efficiency on output.

The vertical axis measures percentage deviations from the path that output would have followed if all the shocks were constant (the random walks would preserve their drifts, but innovations are shut down). The solid line depicts the actual path of output in the data. The rest of the lines depict the paths of output if we shut down innovations to just one of the shocks, eliminating its effect on the economy.

Figure 6 shows that if there were no change in total factor productivity, the recession probably would not have started. The separation shocks added little to the depth of the recession, while shocks to bargaining power and matching efficiency are key to understanding the slow recovery: in the absence of these adverse shocks, the economy would have recovered half the distance to the trend by summer of 2010.

Figure 7 depicts a similar decomposition of unemployment. It follows from this figure that separation shocks are responsible for the initial increase in unemployment. Increases in the bargaining power of workers start playing a role only once the economy is already in a recession, reinforcing this initial increase in unemployment. Declines in matching efficiency leave unemployment at a high level for a longer period

FIGURE 6. Output with all but one shock

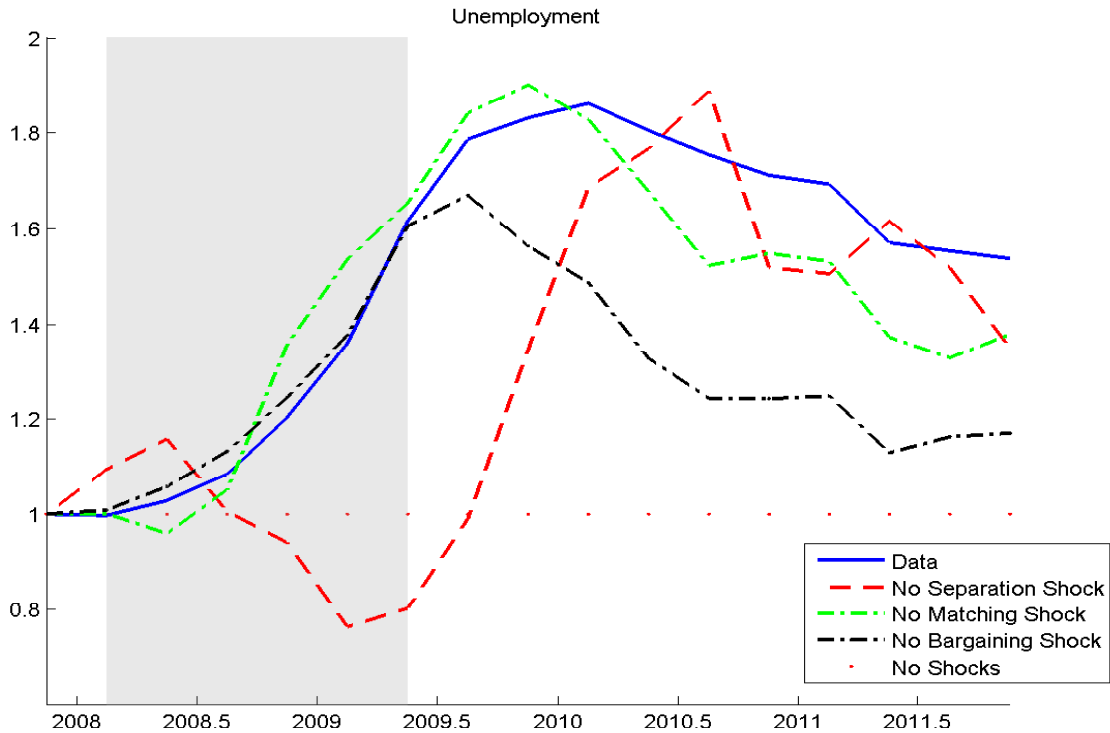


of time after the official recession has already ended, thus accounting for the so-called jobless recovery.

Thus, Figures 6 and 7 demonstrate that although matching shocks explain the largest fraction of the dynamics of the labor wedge, they can only account for a small fraction of output and unemployment dynamics. While shocks to TFP and the separation rate start recessions by accounting for the initial slowdown in output and unemployment, the role of bargaining and matching shocks is to deepen the recession and delay the recovery.

The interpretation of these results is quite clear. After some firms in the economy have become less productive, the role of the separation shock is to create the initial pool of unemployed people. This result is consistent with the role of variations in job destruction and the separation rate emphasized by Mortensen and Pissarides (1994) and Fujita and Ramey (2007).

FIGURE 7. Unemployment with all but one shock



As the number of unemployed goes up, the reservation value of workers goes down significantly—they are willing to work at a lower wage. The sluggish response of wages drives up the bargaining power of the workers, while the firm is now in a worsened position. As a result firms start posting fewer vacancies, and there are more unemployed in the market. Consistent with this explanation, the sharp increase in the bargaining power of workers accounts for the bulk of changes in unemployment and vacancies in the second phase of the recession. This logic is consistent with rigid wages as one of the explanations for variations in unemployment proposed by Hall (2005b).

As the number of workers seeking jobs is high and the number of vacancies is low, the matching efficiency goes down, thus causing output to fall deeper and the recession to last longer. Figure 7 confirms that if there were no decline in matching efficiency, the recovery from the recession would have been faster. Hence, the so-called "jobless recovery" is due largely to matching shocks. We attribute this to

some form of congestion, which still requires an explanation. It can also be some form of disorganization, when the least efficient and more specialized workers become desperate to find a job and wait until better times, consistent with the idea of rest unemployment.

These results are related to the debate between Fujita and Ramey (2007) and Shimer (2005b) on whether job destruction or job creation is more important for fluctuations in unemployment and output. We find that although shocks to job creation are more important for the behavior of output and unemployment, shocks to job destruction cannot be ignored. Changes in the separation rate account for a significant fraction of fluctuations and explain the initial increase in unemployment. Essentially, these shocks start the recession. Thus, even though their contribution to the decline in output is relatively small, without job destruction shocks recessions might not have happened in the first place.

III.4. Decompositions: Summary. Note that the statements made regarding the last recession hold more generally over the period of interest. A similar decomposition of the previous five recession episodes shows that the emphasized pattern holds more generally: separations create the initial pool of unemployed, and adverse matching shocks slow down the recovery.

To summarize contributions of each shock to each variable of interest, we set all the other shocks to their steady-state values and simulate the model. We obtain paths of output, hours, unemployment, vacancies, and the labor wedge, which would have taken place if all the other distortions except one were absent.

Table 5 reports fractions of variations in output, unemployment, hours, vacancies, and the labor wedge, that can be explained by each one of the shocks. Notice that the effect of the labor wedge itself is decomposed into the effects of separation, bargaining and matching shocks. Hence, the total contribution of the "labor wedge" is measured by hitting the economy with all three shocks at the same time.

Table 5 shows that matching shocks are by far the most important for explaining the labor wedge, but the least important for explaining unemployment. On the other hand, separations are nearly twice as important for understanding unemployment as

TABLE 5. Average fractions of variations explained by each shock

Shock	TFP	Invest-t	Gov-t	Separation	Bargaining	Matching
Output	31%	17%	5%	15%	17%	15%
Unemp-t	6%	17%	7%	34%	20%	16%
Hours	11%	20%	22%	8%	24%	15%
Vacancies	5%	13%	6%	27%	38%	11%
L.Wedge				28%	23%	49%

they are for understanding the labor wedge. The fact that matching and separation shocks play such an uneven role in explaining fluctuations in unemployment and the labor wedge, leads us to conclude that the main driving forces behind variations in unemployment are quite different from those behind the labor wedge.

IV. CONCLUSION

Motivated by the fact that variations in the labor wedge account for a large fraction of business cycle fluctuations, some recent papers have pointed out that understanding the labor wedge would give insight into the nature of unemployment. In this paper we ask if labor market frictions that are considered important for explaining unemployment in a search and matching framework are also important for explaining the labor wedge in a business cycle model with search and matching frictions.

Using a model that features time-varying search and matching frictions in the spirit of Mortensen and Pissarides (1994), Shimer (2005a) and Hall (2005c) we decompose the labor wedge into three broad classes of frictions captured by separation, bargaining and matching shocks. Using a business cycle accounting methodology similar to that of Chari, Kehoe, and McGrattan (2007), we identify the driving forces behind variations in the labor wedge and unemployment.

Our main finding is that the forces that drive unemployment are quite different from those that drive the labor wedge. In particular, imperfections in the job destruction and bargaining processes commonly considered in the search literature are not very helpful in explaining the labor wedge but they are important for understanding

unemployment. Likewise, the matching friction is the main driving force behind the labor wedge but it is not nearly as important for unemployment.

This implies that theories emphasizing wage rigidity and endogenous job destruction are not very useful for explaining the behavior of the labor wedge. Instead, according to our results, more attention should be devoted to studying frictions equivalent to the matching shock in our model, for example, frictions that lead to cyclical variations in job creation costs, search effort, or coordination problems. More specifically, one potential microfoundation for the matching shock in our model is proposed by Lester (2010), who shows that when firms have the ability to post multiple vacancies then the efficiency of the matching process depends on the distribution of vacancies among firms, increasing in the concentration of vacancies.

Although matching shocks largely explain the behavior of the labor wedge, both frictions in job creation and job destruction play an important role in unemployment fluctuations. Hence, if the labor wedge is motivated by search and matching frictions, then it is relevant for explaining unemployment mainly through changes in matching efficiency.

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APPENDIX A. APPENDIX. NOT FOR PUBLICATION

A.1. **Appendix A.** In the following subsection we show the solution to the same model as in the main body of the paper but when solved as a social planner's problem. The decentralized version of the model has a missing equilibrium condition that is typically replaced with a Nash bargaining condition to fix the real wage. We take advantage of this missing condition, and by comparing the social planner's solution with the decentralized version of the model, we construct a time-varying bargaining shock, which implicitly determines the wage rate.

A.1.1. *The Social Planner's Problem.* To compare competitive allocations with an efficient one, we solve the social planning problem. The social planner maximizes the discounted present value of the utility function:

$$\max_{\{C_t, L_t, V_t, U_t, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, V_t, U_t) \quad (30)$$

subject to

$$C_t + \frac{1}{T_t} (K_{t+1} - (1 - \delta_K)K_t) + G_t \leq A_t F(K_t, L_t) \quad (31)$$

$$L_t = (1 - \delta_L)L_{t-1} + M_t \quad (32)$$

The optimality conditions of the planner are given by:

$$\frac{1}{T_t} = \beta E_t \frac{U'_{C_{t+1}}}{U'_{C_t}} \left(A_{t+1} F'_{K_{t+1}}(K_t, L_t) + \frac{1}{T_t} (1 - \delta_K) \right) \quad (33)$$

$$A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}} = \Gamma_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \Gamma_{t+1} (1 - \delta_{L_t}) \right) \quad (34)$$

$$-\frac{U'_{V_t}}{U'_{C_t}} = \Gamma_t \frac{\partial M_t}{\partial V_t} \quad (35)$$

$$-\frac{U'_{U_t}}{U'_{C_t}} = \Gamma_t \frac{\partial M_t}{\partial U_t} \quad (36)$$

Together with equations (7)-(8) they describe the allocations a social planner would choose. Γ_t is the Lagrange multiplier associated with the labor accumulation constraint. Given that T_t , A_t , δ_{L_t} , G_t and B_t are exogenous, we have a system of eight equations and eight unknowns $\{K_{t+1}, L_t, C_t, M_t, Y_t, V_t, U_t, \Gamma_t\}$.

A.1.2. *Constructing a Time Varying Bargaining Shock.* By comparing the social planner's optimality conditions with those of the decentralized problem, we can find the necessary assumptions to make the decentralized problem efficient.

By putting equations (12) and (13) together, we get that $-\frac{U'_{V_t}}{U'_{C_t}} = \eta_t \frac{M_t}{V_t}$, and if we compare this expression with equation (35), and equation (14) with equation (36), we need

$$\eta_t \frac{M_t}{V_t} = \Gamma_t \frac{\partial M_t}{\partial V_t} \quad (37)$$

$$\mu_t \frac{M_t}{U_t} = \Gamma_t \frac{\partial M_t}{\partial U_t} \quad (38)$$

so that the optimality conditions on vacancies and unemployment are the same in the decentralized and planner's problem. Furthermore, if we assume that the matching function has constant returns to scale

$$\frac{\partial M(U_t, V_t)}{\partial U_t} U_t + \frac{\partial M(U_t, V_t)}{\partial V_t} V_t = M(U_t, V_t) \quad (39)$$

then $\Gamma_t = \mu_t + \eta_t$ and the decentralized outcome is Pareto optimal. Hence, the Hosios condition for efficiency is given by:

$$\eta_t = \Gamma_t \frac{\partial M_t}{\partial V_t} \frac{V_t}{M_t} \quad (40)$$

$$\mu_t = \Gamma_t \frac{\partial M_t}{\partial U_t} \frac{U_t}{M_t} \quad (41)$$

As an illustration, assume $\frac{\partial M_t}{\partial U_t} \frac{U_t}{M_t} = \theta$ and $\frac{\partial M_t}{\partial V_t} \frac{V_t}{M_t} = (1 - \theta)$, which together with conditions (40) and (41) give

$$\eta_t = (1 - \theta)\Gamma_t \quad (42)$$

and

$$\mu_t = \theta\Gamma_t \quad (43)$$

Notice that if we replace equations (42) and (43) in equations (10) and (11) and sum them up, we get equation (34), hence the optimality conditions for labor in

the decentralized version become equal to the optimality condition for labor in the planner's problem.

Furthermore, if we divide equation (10) by equation (11) and use equations (42) and (43), we get

$$\frac{w_t + \frac{U'_{L_t}}{U'_{C_t}}}{A_t F'_{L_t}(K_t, L_t) - w_t} = \frac{\mu_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \mu_{t+1} (1 - \delta_{L_{t+1}}) \right)}{\eta_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \eta_{t+1} (1 - \delta_{L_{t+1}}) \right)} = \frac{\theta}{1 - \theta} \quad (44)$$

Given that w_t is the wage earned by the worker and $-\frac{U'_{L_t}}{U'_{C_t}}$ is his reservation utility, the term $w_t + \frac{U'_{L_t}}{U'_{C_t}}$ represents the instantaneous benefit from the match earned by the worker. Since the bargaining power of the worker is constant and equal to θ , the optimal wage rate satisfies

$$\left(w_t + \frac{U'_{L_t}}{U'_{C_t}} \right) = \theta \left(A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}} \right) \quad (45)$$

where $A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}}$ is the difference between the marginal product of labor and the marginal disutility of labor. This term represents the instantaneous marginal value of the match, and a fraction θ goes to the worker.

To introduce the time-varying bargaining shock, we build on this result, re-parameterize and substitute θ by ϕ_t . ϕ_t is time-varying and follows an exogenous autoregressive process. Notice that replacing θ by ϕ_t implies that allocations are suboptimal whenever $\phi_t \neq \theta$.

Equations (42) and (43) are replaced by

$$\eta_t = (1 - \phi_t) \Gamma_t \quad (46)$$

$$\mu_t = \phi_t \Gamma_t \quad (47)$$

Once again, if we substitute equations (46) and (47) in (10) and (11), we get equation (34) so it is still true that the optimality conditions for labor of the decentralized version imply the optimality condition for labor of the planner's problem. Dividing

equation (10) by equation (11) and using equations (46) and (47), we get

$$\frac{w_t + \frac{U'_{L_t}}{U'_{C_t}}}{A_t F'_{L_t}(K_t, L_t) - w_t} = \frac{\phi_t \Gamma_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \phi_{t+1} \Gamma_{t+1} (1 - \delta_{L_{t+1}}) \right)}{(1 - \phi_t) \Gamma_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} (1 - \phi_{t+1}) \Gamma_{t+1} (1 - \delta_{L_{t+1}}) \right)} \quad (48)$$

Hence, the optimal wage rate satisfies

$$\left(w_t + \frac{U'_{L_t}}{U'_{C_t}} \right) = \frac{\phi_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \phi_{t+1} \Psi_{t+1} \right)}{1 - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \Psi_{t+1} \right)} \left(A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}} \right) \quad (49)$$

where $\Psi_{t+1} = \frac{\Gamma_{t+1}}{\Gamma_t} (1 - \delta_{L_{t+1}})$ and can be interpreted as a stochastic discount factor for labor.

This wage-setting rule is general enough to capture various mechanisms of wage adjustment, including wage rigidity proposed by Hall (2005a). For example, if we denote $f(\phi_t)$ the first term on the right hand side of equation (49), then the recursive formulation for the bargaining shock

$$f(\phi_t) = (1 - \varrho) \frac{w_{t-1} + \frac{U'_{L_t}}{U'_{C_t}}}{A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}}} + \varrho \theta$$

is equivalent to the partial adjustment wage setting rule proposed by Hall:

$$w_t = (1 - \varrho) w_{t-1} + \varrho w_t^{Nash}.$$

A.1.3. Identification. In this section we show how, given data on allocations (output, investment, consumption, employment, vacancies and unemployment), one can solve for the shocks. Let us first rewrite the equations of the model given the parametric assumptions and functional forms used in the paper:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (50)$$

$$X_t = K_{t+1} - (1 - \delta_K) K_t \quad (51)$$

$$C_t + \frac{X_t}{T_t} + G_t = Y_t \quad (52)$$

$$L_t = (1 - \delta_{L_t}) L_{t-1} + B_t U_t^\theta V_t^{1-\theta} \quad (53)$$

$$\frac{1}{T_t} = \beta E_t \frac{C_t}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{(1 - \delta_K)}{T_{t+1}} \right) \quad (54)$$

$$\mu_t = w_t - C_t \chi (L_t + U_t + V_t)^\gamma + \beta E_t \frac{C_t}{C_{t+1}} \mu_{t+1} (1 - \delta_{L_{t+1}}) \quad (55)$$

$$\eta_t = (1 - \alpha) \frac{Y_t}{L_t} - w_t + \beta E_t \frac{C_t}{C_{t+1}} \eta_{t+1} (1 - \delta_{L_{t+1}}) \quad (56)$$

$$C_t \chi (L_t + U_t + V_t)^\gamma = \eta_t B_t \left(\frac{U_t}{V_t} \right)^\theta \quad (57)$$

$$C_t \chi (L_t + U_t + V_t)^\gamma = \mu_t B_t \left(\frac{V_t}{U_t} \right)^{1-\theta} \quad (58)$$

$$\frac{\eta_t}{\mu_t} = \frac{1 - \phi_t}{\phi_t} \quad (59)$$

Now we shall describe a mechanism to recover the shocks given parameters and functional forms. Given data on consumption C_t (or government spending G_t), output Y_t , investment X_t , employment $L_t + V_t$, number of vacancies V_t and the unemployment rate $\frac{U_t}{L_t + V_t + U_t}$, one can uniquely recover the time path for the variables of interest L_t, V_t, U_t . Then equation (51) uniquely pins down the path for capital given the initial level K_0 , equation (50) pins down the efficiency shock A_t , equation (52) pins down consumption or government spending, and equation (54) can be solved forward to obtain the path for the investment shock as in CKM.

From equations (57) and (58) it follows that $\eta_t U_t = \mu_t V_t$. Then, summing up equations (55) and (56), one obtains:

$$\begin{aligned}
& -C_t \chi (L_t + U_t + V_t)^\gamma + (1 - \alpha) \frac{Y_t}{L_t} \\
& = \mu_t \left(1 + \frac{V_t}{U_t} \right) - \beta E_t \frac{C_t}{C_{t+1}} \mu_{t+1} \left(1 + \frac{V_{t+1}}{U_{t+1}} \right) (1 - \delta_{L_{t+1}})
\end{aligned} \tag{60}$$

Using equation (57) the Lagrange multiplier μ_t can be expressed as a function of the matching shock B_t :

$$\mu_t = \frac{\chi (L_t + U_t + V_t)^\gamma}{B_t \left(\frac{V_t}{U_t} \right)^{1-\theta}} \tag{61}$$

Also the separation rate is connected to the matching shock through the labor accumulation equation (53):

$$(1 - \delta_{L_{t+1}}) = \frac{L_{t+1} - B_{t+1} U_{t+1}^\theta V_{t+1}^{1-\theta}}{L_t} \tag{62}$$

Then, substituting equations (61) and (62) into equation (60), we obtain:

$$\begin{aligned}
& \left(\frac{1 + \frac{V_t}{U_t}}{\left(\frac{V_t}{U_t} \right)^{1-\theta}} \frac{1}{B_t} - 1 \right) L_t = \frac{(1 - \alpha) Y_t}{C_t \chi (L_t + U_t + V_t)^\gamma} \\
& + \beta E_t \frac{C_t}{C_{t+1}} \frac{1 + \frac{V_{t+1}}{U_{t+1}}}{\left(\frac{V_{t+1}}{U_{t+1}} \right)^{1-\theta}} \left(\frac{L_{t+1} + U_{t+1} + V_{t+1}}{L_t + U_t + V_t} \right)^\gamma \left[\frac{L_{t+1}}{B_{t+1}} - U_{t+1}^\theta V_{t+1}^{1-\theta} \right]
\end{aligned} \tag{63}$$

Equation (63) provides a forward-looking equation for the matching shock B_{t+1} as a function of B_t . Solving this equation recursively given some initial value B_0 and making assumptions about expectation formation, we can recover the whole path for the matching shock.¹³ Then equation (62) allows us to back up the separation rate and equations (58) and (57) allow us to calculate the Lagrange multipliers μ_t and η_t . Then, from equation (59), we can compute the bargaining shock ϕ_t .

All together, equations (51-59) describe a one-to-one mapping between the data and the underlying shocks. However the algorithm described here is hard to implement directly for two reasons. First, the equations are forward-looking and can only

¹³In the first step, we assume a diagonal VAR structure for the shocks which allows us to pin down the expectations. We estimate the VAR structure in the second step.

be solved under certain assumptions about expectation formation. Second, many of the parameters of the model are unknown and cannot be simply calibrated from microeconomic data. That is the reason why we postulate stochastic processes for the shocks, linearize the model around a steady-state to compute an approximate solution, and use the Kalman filter to recover the underlying processes for the shocks.

A.2. Appendix B.

A.2.1. *The Detrended Model.* Once we detrend all the variables of the model, we come to the following representation:

$$E_t \psi_{t+1} \left(\alpha \frac{y_{t+1}}{k_{t+1}} - \frac{1 - \delta_K}{\tau_{t+1}} \right) = 1$$

$$y_t = a_t k_t^\alpha L_t^{1-\alpha}$$

$$c_t + z_t k_{t+1} - (1 - \delta_K) \frac{k_t}{\tau_t} + g_t = y_t$$

$$\Gamma_t = \left((1 - \alpha) \frac{y_t}{L_t} - \kappa_t \right) + E_t \psi_{t+1} \Gamma_{t+1} (1 - \delta_{L_t})$$

$$(B_t U_t^\theta V_t^{1-\theta}) \Gamma_t = (V_t + U_t) \kappa_t$$

$$\phi_t V_t = (1 - \phi_t) U_t$$

$$L_t = (1 - \delta_{L_t}) L_{t-1} + B_t U_t^\theta V_t^{1-\theta}$$

$$z_t^{1-\alpha} = a_t \tau_t^\alpha$$

$$m_t = (1 - \delta_{L_t}) \frac{\Gamma_t}{\Gamma_{t-1}}$$

$$\psi_t = \beta \left(\frac{c_{t-1}}{c_t} \right) \frac{1}{z_{t-1}}$$

$$\kappa_t = \chi c_t (L_t + U_t + V_t)^\gamma$$

$$q_t = (1 - \phi_t) \Gamma_t B_t \left(\frac{U_t}{V_t} \right)^\theta$$

$$(1 - E_t m_{t+1} \psi_{t+1}) (w_t - \kappa_t) = (\phi_t - E_t m_{t+1} \psi_{t+1} \phi_{t+1}) \left((1 - \alpha) \frac{y_t}{L_t} - \kappa_t \right)$$

$$X_t = \begin{bmatrix} \log \delta_{Lt} - \log \delta_{Lss} \\ \log \phi_t - \log \phi_{ss} \\ \log B_t - \log B_{ss} \\ \log a_t - \log a_{ss} \\ \log \tau_t - \log \tau_{ss} \\ \log g_t - \log g_{ss} \end{bmatrix}$$

$$X_t = \begin{bmatrix} \rho_S \\ \rho_B \\ \rho_M \\ \rho_A \\ \rho_T \\ \rho_G \end{bmatrix} X_{t-1} + Q \begin{bmatrix} \sigma_S & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_B & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_M & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_A & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_T & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_G \end{bmatrix} \varepsilon_t$$

$$d \log GDP_t = \log \frac{y_t + q_t V_t}{y_{t-1} + q_{t-1} V_{t-1}} z_{t-1}$$

$$d \log Cons_t = \log \frac{c_t}{c_{t-1}} z_{t-1}$$

$$d \log Inv_t = \log \frac{k_{t+1} z_t \tau_t - (1 - \delta_K) k_t}{k_t z_{t-1} \tau_{t-1} - (1 - \delta_K) k_{t-1}} z_{t-1} \tau_{t-1}$$

$$Hours_t = \frac{L_t + V_t}{L_{ss} + V_{ss}}$$

$$Unemp_t = \frac{U_t}{L_t + V_t + U_t}$$

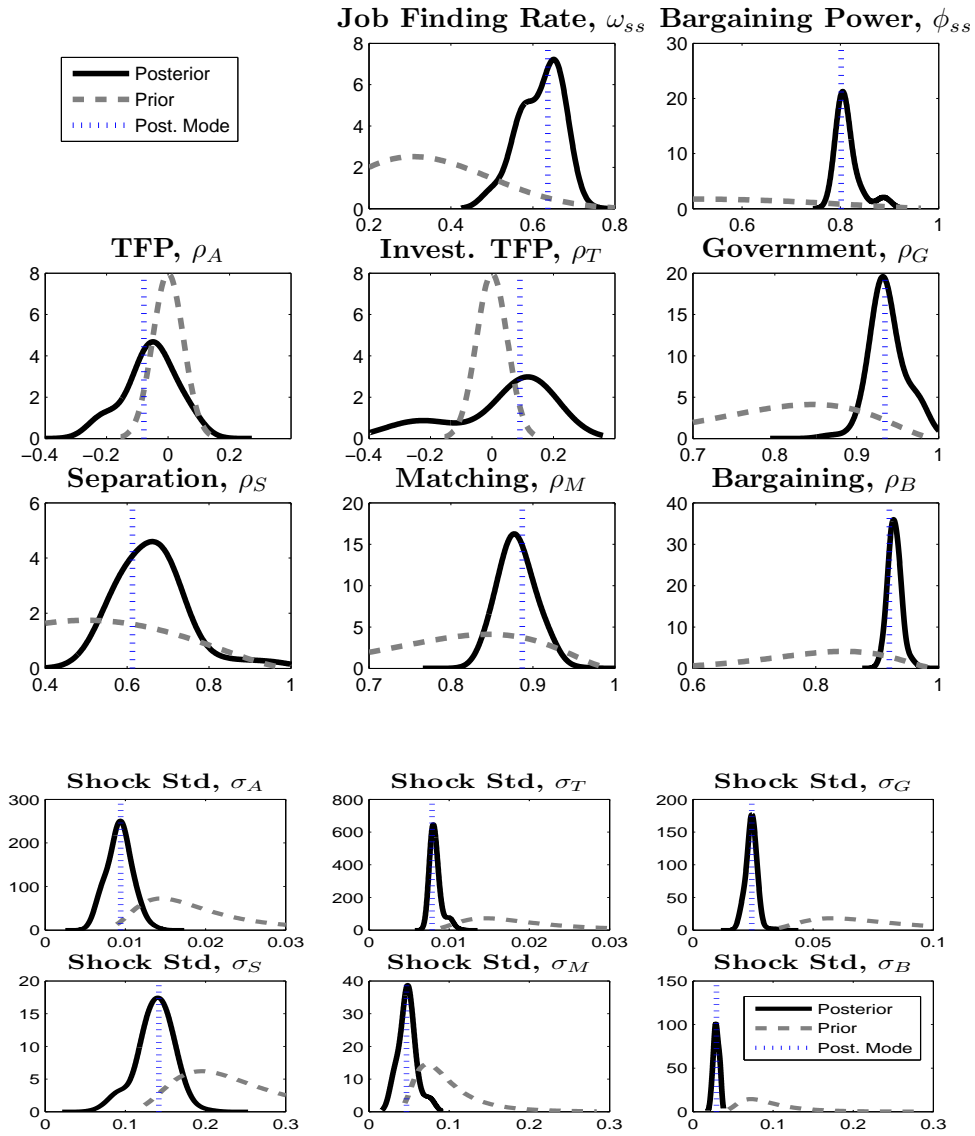
$$HWant_t = \frac{V_t}{V_{ss}}$$

A.2.2. *Computing the Steady-State.* Choose a value of L_{ss}

- 1) $z_{ss} = (a_{ss}\tau_{ss}^\alpha)^{\frac{1}{1-\alpha}}$
- 2) Denote $\varphi = \left(\left(\frac{z_{ss}}{\beta} + \frac{1-\delta_K}{\tau_{ss}} \right) / \alpha a_{ss} \right)^{-\frac{1}{1-\alpha}}$
- 3) $k_{ss} = \varphi L_{ss}$ $y_{ss} = a_{ss}\varphi^\alpha L_{ss}$
- 4) $c_{ss} = \left[(1 - g_{ss}) a_{ss}\varphi^\alpha - \left(z_{ss} - \frac{(1-\delta_K)}{\tau_{ss}} \right) \varphi \right] L_{ss}$
- 5) $B_{ss} = \frac{1}{\omega_{ss}} \left(\frac{\phi_{ss}}{1-\phi_{ss}} \right)^{1-\theta}$
- 6) $U_{ss} = \omega_{ss}\delta_L L_{ss}$ $V_{ss} = \frac{1-\phi_{ss}}{\phi_{ss}} U_{ss}$
- 7) $\xi = \frac{y_{ss}}{L_{ss}c_{ss}} \frac{1-\alpha}{\left(1 + \frac{\omega_{ss}}{\phi_{ss}} \left(1 - \frac{\beta}{z_{ss}} (1-\delta_L) \right) \right)}$
- 8) We have assumed a normalization $\chi = \frac{\xi}{(L_{ss} + U_{ss} + V_{ss})^\gamma}$
- 9) $\kappa_{ss} = \xi c_{ss}$ $m_{ss} = 1 - \delta_L$
- 10) $\Gamma_{ss} = \xi c_{ss} \frac{\omega_{ss}}{\phi_{ss}}$
- 11) $w_{ss} = \phi_{ss} (1 - \alpha) \frac{y_{ss}}{L_{ss}} - (1 - \phi_{ss}) \kappa_{ss}$
- 12) $\psi_{ss} = \frac{\beta}{z_{ss}}$ $q_{ss} = \kappa_{ss}$

A.3. **Appendix C.**

FIGURE 8. Prior (grey) and posterior (black) distributions of parameters



A.4. Appendix D.

A.4.1. Comparison to CKM.

A.4.2. Full Decompositions and Correlation Structure. Table 6 reports the numbers from the original paper by Chari, Kehoe, and McGrattan (2007). Comparing the second row of Tables 6 and 7 one can verify that our decompositions are comparable with those of CKM since the difference in the contributions of TFP, investment

FIGURE 9. Comparison of the Labor Wedge to the Estimate of CKM

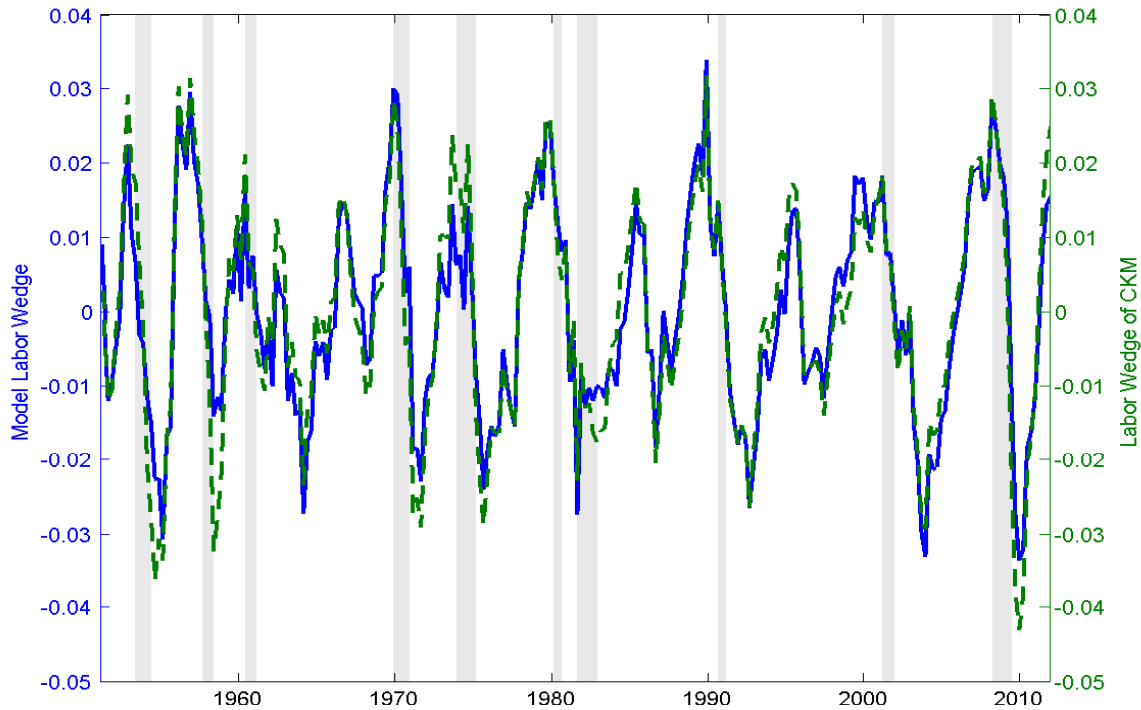


TABLE 6. Ratios of standard deviations explained by each wedge.

Source: Chari, Kehoe and McGrattan (2007)

Wedge	Efficiency	Investment	Labor
Output	0.73	0.31	0.59

and labor shocks is insignificant. Table 7 also gives a clearer picture of the relative contributions of the labor shocks.

Table 8 reports the same fractions of standard deviations as Table 5, but averaged over a selection of recession periods. It demonstrates that during recessions the labor wedge and TFP play a slightly more important role in business cycles than in normal times, while the contribution of investment shocks is negligible both in recessions and overall.

TABLE 7. Fractions of variations explained by each shock over the whole period

Shock	TFP	Investment	Government	Labor
Output	31%	17%	5%	47%
Consumption	23%	22%	15%	40%
Investment	31%	29%	10%	30%
Unemployment	6%	17%	7%	70%
Hours	11%	20%	22%	47%
Vacancies	5%	13%	6%	76%

Shock	TFP	Separation	Bargaining	Matching
Output	31%	15%	17%	15%
Consumption	23%	2%	13%	25%
Investment	31%	7%	17%	6%
Unemployment	6%	34%	20%	16%
Hours	11%	8%	24%	15%
Vacancies	5%	27%	38%	11%

Table 9 reports cross correlations of shocks at different lags confirming the picture of TFP and separation shocks starting recessions and bargaining and matching shocks coming into play only later on.

A.5. **Appendix E.** Figure 10 demonstrates the emphasized decomposition of unemployment for the previous four recession episodes: separations create the initial pool of unemployed, and adverse matching shocks slow down the recovery.

Figure 11 shows that if there was no change in the labor wedge, the recession would have been much shorter (if at all noticeable) and half as severe. If there was no change in total factor productivity, the recession probably wouldn't have started. An absence of investment shocks would have almost no effect on the path of output. Thus the TFP shock is at work mostly at the start of the recession of 2001. The labor wedge explains the bulk of fluctuations in output after the recession has started.

TABLE 8. Ratios of standard deviations explained by each shock averaged over 6 recessions (70,75,82,91,01,08)

Shock	TFP	Investment	Government	Labor
Output	31%	10%	7%	52%
Unemployment	6%	15%	7%	72%
Hours	8%	13%	13%	66%
Vacancies	5%	11%	6%	78%

Shock	TFP	Separation	Bargaining	Matching
Output	31%	8%	30%	14%
Unemployment	6%	31%	27%	14%
Labor Wedge		25%	32%	43%
Hours	8%	11%	37%	18%
Vacancies	5%	22%	46%	10%

TABLE 9. Cross Correlations of Lags and Leads of Shocks

Shocks (X,Y)	Correlation of X with Y at lag k				
	-2	-1	0	1	2
TFP, Investment	-0.19	-0.08	-0.24	0.32	0.08
TFP, Government	0.17	0.13	0.10	-0.01	-0.06
Investment, Government	-0.09	-0.17	-0.23	-0.25	-0.24
TFP, Separation	0.36	0.22	-0.02	-0.27	-0.27
TFP, Bargaining	0.37	0.36	0.24	0.12	0.00
TFP, Matching	-0.19	-0.13	0.02	-0.12	-0.09
Separation, Bargaining	0.15	0.38	0.59	0.69	0.68
Separation, Matching	0.00	-0.09	-0.29	-0.18	-0.17
Bargaining, Matching	-0.52	-0.57	-0.55	-0.49	-0.42

FIGURE 10. Effects of separation and matching shocks on unemployment

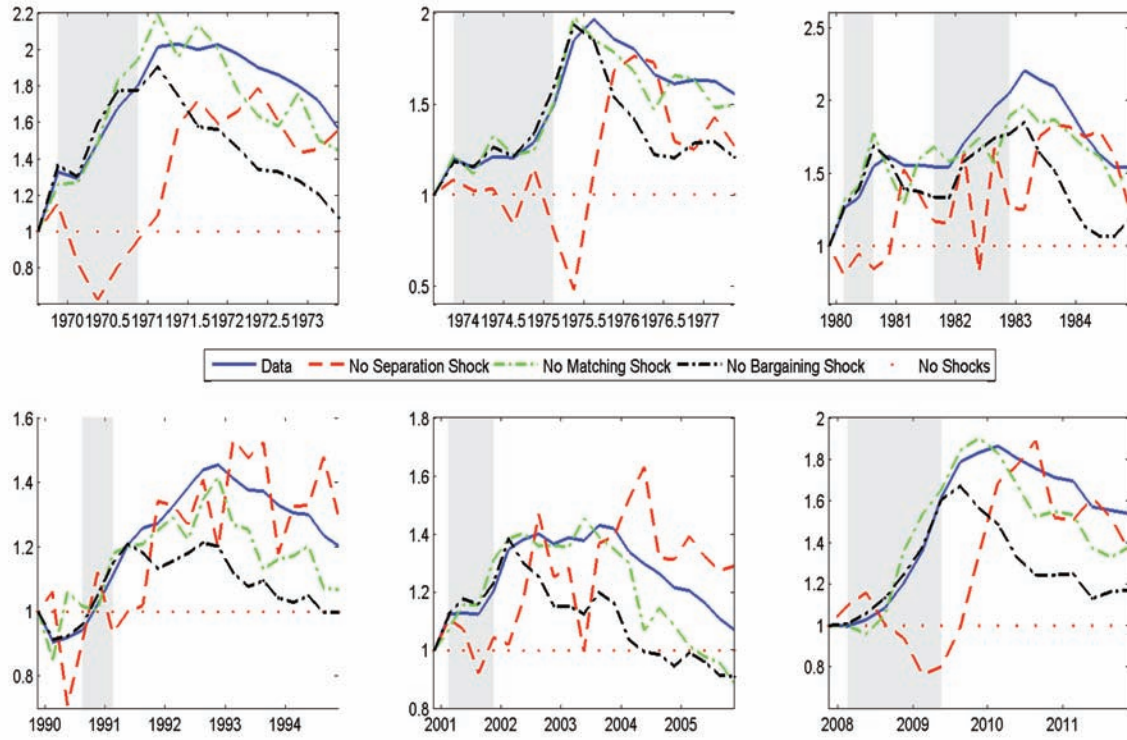


FIGURE 11. Output with all but one shock

