Macroeconomic Volatility: The Role of the Informal Economy

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Abstract

Many countries have a large informal economy that is poorly measured in the national accounts. I develop a two-sector small open economy business cycle model where one sector is formal and the other is informal, and explore the effect that the informal sector has on measured business cycles. I show that if the informal economy is poorly measured, the model can generate a volatility of measured consumption that is higher than that of output, as observed in many developing countries and some developed countries, even though actual consumption is not nearly as volatile. My results illustrate the importance of the informal sector and its mis-measurement in understanding measured cyclical fluctuations.

JEL: E26, E01, E32, E41

Keywords: Informal Economy; Business Cycles; Relative Volatility; Mis-measurement; Calibration.

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1 Introduction

In many countries (particularly developing countries) there is a large informal economy that is poorly measured in national account statistics.[1] However, very little or no attention has been paid to how this mis-measurement of the informal sector affects the behavior of business cycle fluctuations. In this paper I develop a two-sector small open economy business cycle model where one sector is formal and the other is informal, and explore the effect that the informal sector has on measured business cycles.

I show that if the informal sector is poorly measured, this model can account for a business cycle pattern that is puzzling from the standpoint of standard business cycle models. In particular, as opposed to a standard one-sector business cycle model featuring consumption smoothing, this model can generate a volatility of measured consumption that is higher than that of output. Interestingly, this means that the model can rationalize high observed volatility of measured consumption relative to output in both developing countries and some developed countries that have a large informal economy. These results illustrate the importance of recognizing the informal sector in order to understand business cycle fluctuations. This excess volatility of consumption puzzle that is observed in the data is well known in the business cycle literature and has been documented by Rand and Tarp (2002), Mendoza (1995), Neumeyer and Perri (2005), and Aguiar and Gopinath (2007), among others.

A large informal sector, which is poorly measured due to its underground nature, can be reflected in a volatility of consumption that is greater than that of output because in response to a relative productivity shock, agents can substitute formal for informal consumption over the cycle and we mostly observe movements in and out of the formal sector. Furthermore, if we mostly observe movements in the formal sector and only formal output can be traded, then consumption can be more volatile than output because in response to a relative productivity shock, formal output does not respond as much as formal consumption.

I consider a model where a representative agent can decide whether to work in the formal sector and pay income taxes, or work in the informal sector where she does not have to pay taxes, but faces a risk of a tax audit and a punishment if she is found evading taxes. The informal sector faces a productivity shock; the agent consumes both formal and informal consumption goods and invests in international capital markets.[2] The government collects the labor tax from the formal sector and

[1] I define the informal economy as those market-based value-added creating activities which are not taxed or registered by the government. In developing countries the informal sector has been estimated to be around 36% of GDP. In developed countries it has been estimated to be around 13% of GDP. However, there is a subset of developed countries for which it is considerably higher, at around 20% of GDP (see Schneider (2007)).

[2] In this model because relative productivity is what matters, having a relative productivity shock in the informal sector is analogous to having both the formal and the informal sector face a productivity shock that is positively but not perfectly correlated due to financial frictions faced by the informal sector, for example.

uses these resources to improve law enforcement.

If the formal and informal sectors are well measured (so that output and consumption are given by the sum of formal and informal output and consumption), when the informal sector receives a good productivity shock, agents substitute formal for informal consumption.[3] As consumption goods are negatively correlated between the two sectors, the model implies that total consumption has a low volatility. At the same time, due to a strategic savings motive, the trade balance is not as volatile as formal consumption, so the covariance between formal and informal output is lower than that of consumption. As a result, total consumption is less volatile than total output as in the standard one-sector model.

However, if the formal sector is measured but the informal sector is not (meaning that measured output and consumption are just formal output and consumption), when the informal sector receives a good productivity shock, agents substitute formal for informal consumption, but this time, as only the formal sector is measured, we observe a highly volatile consumption. Furthermore, when the informal sector is not measured, consumption can be more volatile than output because the substitution of consumption goods from the formal to the informal sector over the business cycle is greater than that of the change in net foreign assets (because of a strategic savings motive).

I parametrize the model to assess the quantitative importance of these mechanisms. In my numerical exercise I study how the relative volatility of consumption to output changes when the size of the informal economy increases. Given the uncertainty of the degree to which the informal economy is measured in the national account statistics in practice, I do the exercise both when the informal sector is fully measured in the model and in the extreme case when it is not measured at all.

I find that theory predicts that the volatility of consumption is greater than that of output in countries with large *unmeasured* informal sectors. My results imply that if countries could accurately measure the informal sector in their national accounts, then their observed relative volatility of consumption to output would fall significantly. These results suggest that a mis-measured informal sector can be very important in accounting for observed business cycle fluctuations.

This paper follows the home production literature, the literature on emerging market business cycles and the literature on informality. As in home production models, volatility arises due to relative productivity differentials between the formal and the informal sector (see Benhabib et al. (1991), pg. 1168). Nevertheless, the informal sector is different from the household sector because the former is market-based while the latter is not. Consequently, informal production should ideally be included in the national accounts while home production should not.

[3]Formal and informal consumption goods are substitutes because the fruit and/or vegetable that is sold by a vendor on the street is the same one that is found in the supermarket.

In terms of the emerging market business cycles literature, this paper is more closely related to Neumeyer and Perri (2005) and Aguiar and Gopinath (2007), who study the excess relative volatility of consumption to output. Neumeyer and Perri construct a model of a small open economy where the real interest rate is decomposed in an international rate and a country risk component; the key features of the model are that country risk is affected by fundamental shocks and that there is working capital. They find that if interest rate shocks are large compared to productivity shocks, their model induces equilibrium consumption to be more volatile than output. Aguiar and Gopinath propose a small open economy model which is subject to volatility in trend growth due to frequent policy regime switches. They find that shocks to trend growth are the primary source of fluctuations in emerging markets. Their model predicts that consumption volatility (driven by income shocks) should exceed income volatility in emerging economies. The "informal sector" channel presented in this paper, is thus a mechanism driving high consumption volatility in emerging markets that is complementary to other mechanisms in the literature. Nevertheless, unlike mechanisms in previous literature, the mechanism in this paper not only explains the excess relative volatility of consumption to output that we observe in emerging markets but also explains the high consumption volatility observed in some developed countries.

The paper relates to the literature that is devoted to measuring the informal economy through different methods. In Section 2, I explain and use the results from the Currency-Demand approach as well as the DYMIMIC method proposed by Schneider and Enste (2000) and Schneider (2007).

Finally, although the purpose of the paper is not to construct a novel model that gives rise to the informal sector, in this model agents choose how much time to spend in the formal versus the informal sector, making the size of the informal sector endogenous. To that extent it is related to Quintin (2008), where the size of the informal sector decreases as the degree to which financing contracts can be enforced in the formal sector rises. In the present model there is no lack of enforcement, but a financial friction of this sort is represented by changes in the average relative productivity between the formal and the informal sector. Also, when deciding how much time to spend in one sector versus the other, agents face a tradeoff between paying taxes and running the risk of an audit, as in Quintin (2008) and Aruoba (2010). And in line with the findings in Aruoba (2010), in this model as a consequence of a larger informal sector, the rule of law in the economy deteriorates.

The rest of the paper proceeds as follows: Section 2 shows the relationship between the relative volatility of consumption to output and the size of the informal economy, Section 3 describes the model economy, Section 4 explains the calibration and computation of equilibrium, Section 5 shows the results, and the last section summarizes my findings.

2 Relative Volatility of Consumption to Output and the Informal Economy

In this section I begin by defining the term *informal economy*. I discuss two methods that are commonly used to estimate its size and the average results obtained by applying these methods to developed and developing countries. These results, not surprisingly, establish that rich/developed countries have smaller informal economies than poor/developing countries. Nevertheless, they also show that there is a subset of developed countries which have a large informal sector. I then discuss how incorporating the informal economy when compiling the national accounts is a difficult task due to its underground nature. The section ends by showing that there is a systematic relationship between volatility and the size of the informal sector: the relative volatility of consumption to output ($\frac{\sigma(c)}{\sigma(y)}$, the ratio of their standard deviations) is higher in countries with large informal sectors.

I define the informal economy as those market-based value-added-creating activities which are not taxed or registered by the government and are therefore difficult to measure.[4] There are several ways of indirectly measuring the size of the informal economy. The currency-demand approach assumes that informal activities operate with cash, and it calculates the correlation between currency demand and "tax pressure." If the tax burden and the demand for money increase together then the increased demand for money is attributed to an increase in the size of the informal economy (see Schneider and Enste (2000)). Another approach is the dynamic multiple-indicators multiple-causes model (DYMIMIC), which considers multiple causes and indicators of the informal economy at the same time. This approach estimates a model in which the informal economy is an unobserved variable. The model links this unobserved variable to observed indicators (like electricity use, for example). The appendix contains a more detailed description of commonly used indicators. For details on the DYMIMIC approach, refer to Schneider and Enste (2000). Even though each approach has its deficiencies,[5] each methodology used to measure the informal sector allows comparison across countries. In other words, if we use the same methodology on all countries, we can compare the estimates of the size of the informal economy among them. Furthermore, in order to be more agnostic towards the different methodologies to measure the informal sector, the measures shown in this section are an average of the results obtained using the currency-demand approach and the DYMIMIC approach (see Schneider (2007)).

Schneider (2007) shows that in rich countries the informal sector is between 8% and 23% of GDP while in poor countries it is between 20% and 60% of GDP. Nevertheless, most developed countries have an informal sector that is between 8% and 17% of GDP, while Scandinavia (Swe-

[4]Note that this definition explicitly excludes home production as it is not market-based.

[5]See Appendix for a discussion on the pros and cons of different methodologies used to measure the informal economy.

den, Norway, and Denmark), Spain, and Portugal have an informal sector that is between 17% and 22% of GDP. This last group of developed countries has an informal economy that is larger than that of the average developed country and that can be of the same size as that of some developing countries. Later, I will show that the size of the informal economy is not the only similarity between Scandinavia, Spain, and Portugal and developing countries; they also share a higher relative volatility of cyclical consumption.

Most countries follow the methodology suggested in the book *"System of National Accounts, 1993 (SNA93)"* OECD (2000) to measure macroeconomic variables. This methodology requires that they calculate the income side of the accounts using value added and then measure the expenditure side by only calculating investment, government expenditure and net exports. Hence, consumption is a residual. OECD countries (or any country for that matter) can complement the SNA93 methodology with that suggested in *"Measuring the Non-Observed Economy–A Handbook"*, published by the OECD (2002). This book gives a detailed decomposition of the informal economy (which the book refers to as the non-observed economy) and then it describes several methodologies that can be used to identify and quantify the informal economy so that it can be incorporated into the national accounts. The most recommended approach is to use household surveys.

Through a household survey it is possible to measure income and expenditure. The resulting discrepancy attained by comparing these results with those obtained by following the SNA93 approach might point out production and consumption of informal goods and, hence, the adjustments that need to be made to the accounts. Another approach is to use the household survey to pin down the labor input used in the informal sector. To do so, one can compare the number of jobs declared by firms with the number of employed people found through the household survey. The number of employed people exceeding the number of jobs represents the informal workforce. Once the informal number of workers is identified, informal workers can be attributed the same gross compensation (net of social contributions) as similar, formal workers.[6] These two methods do not address the informal economy directly, so the results cannot be distinguished from statistical discrepancy and are subject to willingness to report issues as discussed later.

If household surveys are directly used to uncover the size of the informal sector, the results will depend directly on the questions asked by the survey and the definition of the informal economy that is used. For example, the informal sector may be defined as those people who do not contribute to a pension fund, but this definition clearly excludes several important elements that would describe the informal economy otherwise. Another common definition is that a person is considered to work in the informal economy if they work for a firm that has N or fewer workers. This is a very strong assumption because a firm can be very small but still comply with the law. That is, a firm's

[6]This is the approach used in Italy (see Bovi (2007)).

production can be reported to the authorities, meaning that its value added already appears in the GDP no matter how small the firm is. On the other hand, if a direct questionnaire is used, people are usually unwilling to admit (unwilling to report) that they are informal workers, either because they feel afraid of getting caught or because they feel ashamed. This makes it difficult to estimate the extent of undeclared work. The disadvantages of this method are discussed in great detail by Mogensen et al. (1995). As a result, it is natural to think that countries with larger informal sectors are missing a bigger fraction of it in the national accounts.

Many poor and middle-income countries either do not have household surveys or have surveys with poor coverage (they only cover the main metropolitan areas). Hence, they cannot use these surveys to help adjust their national accounts for the informal economy. To the extent that these surveys can be used, they only capture the informal economy of the areas included in the survey.[7] It is more difficult for poor and middle-income countries to correct their accounts for their much larger informal economies.[8]

In summary, accounting for the informal economy in the national accounts is difficult. Developed countries devote more resources to its measurement than developing countries; but, no matter what the efforts to measure it are, countries are missing a big slice of the pie. This mismeasurement is especially relevant for those countries with large informal sectors.

I use an average of the currency-demand and DYMIMIC approach results to compare the size of the informal sector among various countries and to show its relationship to the relative volatility of consumption to output. Table 1 shows the average size of the informal sector[9] and the average relative volatility of consumption to output for thirteen developing and fourteen developed countries.[10] Developed countries are separated into two groups: Scandinavia, Spain, and Portugal and all the rest. The table illustrates how, on average, the higher the relative volatility of consumption to output is, the larger the size of the informal economy. More surprisingly, it shows that, on average, Scandinavia, Spain, and Portugal have a relative volatility of consumption to output that is greater than one, as is the case for developing economies. Hence Scandinavia, Spain, and Portugal have a larger informal economy than the other developed countries and share a higher volatility of

[7] I thank Jaime Vallecilla, Mario Nigrinis, and José Ignacio Lopez for providing information on the measurement of the national accounts in developing countries.

[8] A middle-income country that is an exception is Peru. Peru has been using methods to improve the measurement of its informal sector since the 1980's and has been doing it in a continuous manner since 2003. See http://www.inei.gob.pe/biblioineipub/bancopub/Est/Lib0166/Libro.htm for further information. This observation goes in line with my theory that mis-measurement causes high relative volatility of consumption to output. Peru is the only developing country in the sample with a relative volatility of consumption to output below one.

[9] The estimates in the table correspond to the 2002-03 period; there are also estimates for the 1999-00 and 2001-02 periods, but they are not significantly different (seeSchneider (2007)).

[10] The countries in the sample are the same as in Aguiar and Gopinath (2007) plus the U.S. The countries are middle-income and developed economies that have at least 40 quarters of data, excluding all G-7 countries other than Canada and the U.S. The countries in Neumeyer and Perri (2005) are a subset of those in Aguiar and Gopinath (2007).

cyclical consumption with developing countries.

Figure 2.1, constructed with data from Schneider (2007), Neumeyer and Perri (2005), and Aguiar and Gopinath (2007), shows a scatter-plot of the size of the informal economy and the relative volatility of consumption to output for the same sample of countries in Table 1. The important thing to notice is that there is a high correlation between the relative volatility of consumption to output and the size of the informal sector.[11] In general, countries that have a higher relative volatility of consumption to output have a large informal sector that is difficult to measure.

Table 1: Comparison of the average relative volatility of consumption to output and the average size of the informal economy (% of GDP, 2002-03)

	$\frac{\sigma(c)}{\sigma(y)}$	**Informal Economy (% of GDP)**
Developed Countries	0.81	13
Scandinavia, Spain & Portugal	1.20	20
Developing Countries	1.53	36

Source: Schneider (2007), Neumeyer and Perri (2005) and Aguiar and Gopinath (2007).

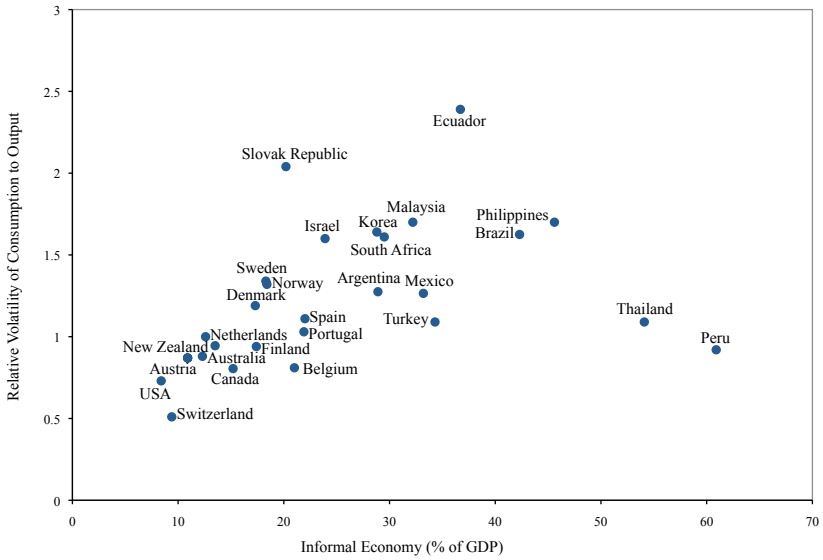
Motivated by these facts, I develop a model to study how the size of the informal sector and its measurement in the national accounts impacts the relative volatility of consumption to output. I do a quantitative analysis to see how the relative volatility of consumption to output changes as the size of the informal economy increases, both when this sector is fully measured and in the extreme case that it is not measured at all.

3 The Model Economy

I develop a parsimonious small open economy model of the informal sector where a representative agent can decide whether to work in the formal sector and pay income taxes, or work for the informal sector where she does not have to pay taxes, but faces a risk of a tax audit and a punishment if she is found evading taxes. The agent consumes both formal and informal consumption goods and invests in international capital markets. The government collects the labor tax from the formal sector and uses these resources to improve law enforcement, i.e. perform tax audits.

[11] The correlation is about 0.75.

Figure 2.1: Relative volatility of consumption to output versus the size of the informal economy measured as % of GDP for the period 2002-03 using the currency demand and model approach.



Source: This plot was constructed with data from Schneider (2007), Neumeyer and Perri (2005) and Aguiar and Gopinath (2007).

Representative Household

The representative agent can work in the formal sector (n_t^f) and pay labor taxes (τ) on their income (w_t), or operate an informal technology (y_t^i), which provides them with $c_t^i = y_t^i$ units of informal consumption with probability $(1-p_t)$. This means that with probability p_t, the government will audit the agent and she will have to pay a tax equal to that paid in the formal sector (τ) plus a punishment ϕ on informal production. The problem of the representative household is to maximize the discounted expected value of its lifetime utility subject to its budget constraint:

$$\max_{\{c_t^f, c_t^i, n_t^f, a_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^f, c_t^i)$$

subject to

$$c_t^f + q_t c_t^i + a_{t+1} = p_t(1-\phi\tau)q_t y_t^i + (1-p_t)q_t y_t^i + (1-\tau)w_t n_t^f + (1+r_t)a_t + T_t,$$

and such that:

$$q_t c_t^i = (1 - p_t \phi \tau) q_t y_t^i, \tag{3.1}$$

$$a_t \geq \bar{a}, \tag{3.2}$$

$$0 \leq n_t^f \leq 1,$$

and

$$c_t^f \geq 0,\ c_t^i \geq 0,$$

where $\beta \in (0,1)$, c_t^f denotes consumption of formal goods, c_t^i denotes consumption of informal goods and q_t is the relative price of informal consumption. Equation 3.1 states that informal consumption is equal to the expected informal production. Apart from deciding how much to consume, and how much time to spend in the formal sector, the representative household decides its net foreign asset position for next period, a_{t+1} subject to an exogenous financial constraint given by Equation 3.2, and receives a lump sum transfer from the government, T_t.

Formal Sector

The formal sector is competitive. Producers maximize profits by choosing the level of capital and labor given the international rental rate of capital and the real wage. They operate using a standard Cobb-Douglas production function. The problem faced by the formal firms is:

$$\max_{\{k_t, n_t^f\}} \Pi_t^f = {k_t}^{\alpha} {n_t^f}^{(1-\alpha)} - r_t k_t - w_t n_t^f - \delta k_t \ \forall t, \tag{3.3}$$

where k_t is formal capital demand, r_t is the international rental rate of capital, and w_t is the wage in the formal sector. The first order conditions for the producers of the formal good are standard.

Informal Sector

The representative agent is endowed with one unit of time which can be divided between formal and informal activities.[12] Thus, if we let n_t^f denote the fraction of time spent working in the formal sector, then $n_t^i = 1 - n_t^f$ is the amount of time spent in the informal sector.

Informal production is given by $y_t^i = z_t (n_t^i)^{\nu}$, where z_t is relative informal productivity and it follows a discrete Markov chain, with an autoregressive coefficient ρ_z and a standard deviation σ_z.

Note that, as in the household production literature, labor is free to flow between sectors. This free mobility assumption is consistent with evidence from the literature on labor market segmentation between the formal and informal sector. Magnac (1991), using an integrated microeconomic

[12] Note that leisure is not valued, so the agent will devote the whole unit of time to productive activities.

model derived from a labor supply model in a four-sector labor market with explicit demand constraints and using data on women's labor force participation in the main towns of Colombia in 1980, concludes there is free mobility within the labor market. Maloney (1999), using detailed panel data from Mexico, studies patterns of worker mobility between sectors and concludes that there is no significant segmentation between the formal and informal division. Finally, Pratap and Quintin (2006), using parametric and semi-parametric methods and data from Argentina's household survey, find that there is no evidence to support "the mainstream view that labor markets are segmented along formal/informal lines in developing nations such as Argentina."

Government

The government taxes formal labor income and at the end of the period rebates formal labor taxes to households as a lump sum transfer:

$$\tau w_t n_t^f = T_t.$$

The government is unable to observe informal activities, so in order to improve the rule of law, it uses tax revenue to increase monitoring by conducting tax audits and uncovering informal production. As a result, the probability of being audited and caught operating the informal technology is given by:

$$p_t = G_t (n_t^i)^\gamma, \tag{3.4}$$

where

$$G_t = \tau w_t n_t^f + p_t \phi \tau y_t^i. \tag{3.5}$$

Note that G_t is total tax revenue; $\tau w_t n_t^f$ which represents tax revenue coming from the formal sector and $\phi \tau y_t^i$ which is collected from the informal sector with a probability p_t. Furthermore, Equation 3.4 implies that the probability of getting caught increases with the size of the informal sector, but it also increases with tax revenue, and at the same time informal tax revenue will increase with p_t; so, Equations 3.4 and 3.5 are jointly determined.

Note that the government only rebates to households the tax revenue from the formal sector. As in Bassetto and Phelan (2008) and Aruoba (2010), whenever the government audits an informal worker and collects $\phi \tau y_t^i$, this becomes a "wasteful" punishment or "wasteful" government spending denoted by $g_t = p_t \phi \tau y_t^i$.

Competitive Equilibrium

The agent's problem can be written as a dynamic programing problem. Since the labor allocation problem is static, it can be solved by:

$$\max_{n_t^f \in [0,1]} p_t(1-\phi\tau)q_t z_t(1-n_t^f)^{\nu} + (1-p_t)q_t z_t(1-n_t^f)^{\nu} + (1-\tau)w_t n_t^f,$$

which yields an optimal formal labor-supply allocation n^*.

Thus, an agent's total (formal plus informal) earnings function is given by:

$$y(z_t, w_t) = p_t(1-\phi\tau)q_t z_t(1-n^*)^{\nu} + (1-p_t)q_t z_t(1-n^*)^{\nu} + (1-\tau)w_t n^*,$$

and the dynamic programming problem is the following:

$$V(z,a) = \max_{c^f, c^i, a'} \left\{ u(c^f, c^i) + \beta E\left[V\left(z', a'\right)\right] \right\} \tag{3.6}$$

such that

$$c^f + qc^i + a' = y(z,w) + (1+r)a + T, \tag{3.7}$$

$$c_t^f \geq 0,\ c_t^i \geq 0 \tag{3.8}$$

$$a_t \geq \bar{a}, \tag{3.9}$$

$$c_t^f = y_t^f + (1+r_t)a_t - a_{t+1}, \tag{3.10}$$

$$c_t^i = (1-p_t\phi\tau)y_t^i, \tag{3.11}$$

and the aggregate budget constraint $c + a' + g_t - (1+r)a = y^i + y^f$ is satisfied.

Equations 3.10 and 3.11 represent the market clearing conditions for the formal and informal sectors respectively. The market clearing condition for the formal sector states that formal production can be used for consumption or to save in the international capital market (only formal goods are traded). The market clearing condition for the informal sector states that informal consumption is equal to informal production minus the fraction that is seized with probability p.

I solve for the stationary equilibrium ($w_t = w$, $r_t = r$, $q_t = q$), which consists of prices r, w, and q; value function, $V(z,a)$; optimal decision rules, $c^f(z,a)$, $c^i(z,a)$, $a'(z,a)$; assets and capital stock a, k; and labor supply in both formal and informal sectors, n and $1-n$, such that agents optimize given prices (r,w,q) and government policies (τ, T). The value function, $V(z,a)$, is a solution to Equation (3.6) subject to (3.7)-(3.11), and $c^f(z,a)$, $c^i(z,a)$, and $a'(z,a)$, are the associated opti-

mal decision rules. Finally prices r, w satisfy the firm's optimality conditions and the government satisfies its budget constraint.

4 Calibration and Calculation of Equilibrium

In order to compare the model-generated relative volatilities with the observed ones, we want to study what happens as the size of the model-generated informal sector increases. I calibrate the model for four different stationary equilibria that resemble the conditions of four of the countries found in Figure 2.1. This countries are Austria, Spain, South Africa, and Philippines. I chose these countries because their informal sector is of different size (ranging from 10% of official GDP for Austria to 45% for Philippines) and they all have the data availability for calibration and for an outside check of the results.

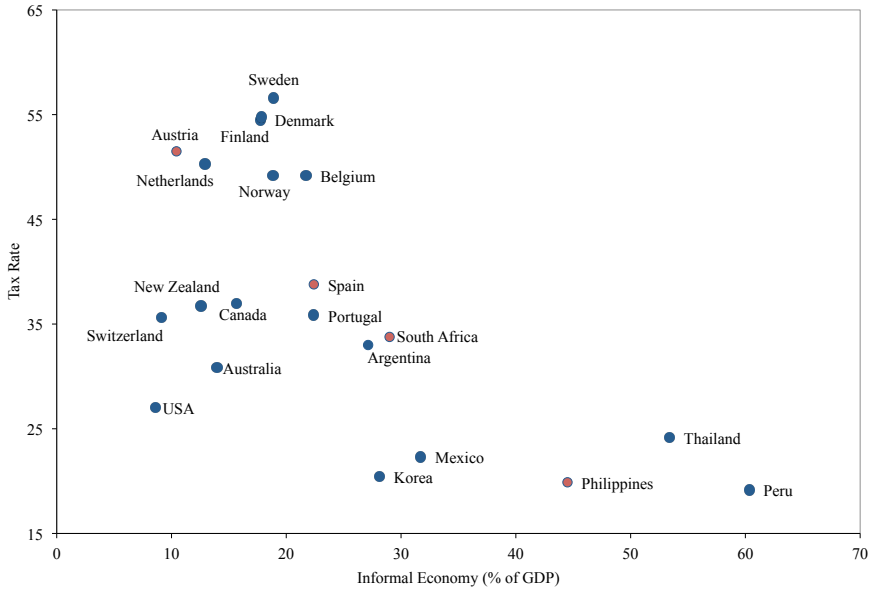
There are certain parameters that are standard in the business cycle literature or in the small open economy literature that are kept constant throughout the four equilibria. These are the capital intensity, the depreciation rate, the international interest rate, the discount factor, and the coefficient of risk aversion. Capital intensity in the formal sector is standard and was set at $\alpha = 0.32$. The depreciation rate was set at 10% per year. The international interest rate was set at 4% per year ($r = 0.04$), and as this is a small open economy model the discount factor has to be equal to the inverse of the international interest rate, $\beta = \frac{1}{1+r}$. I set the coefficient of risk aversion in the utility function to be $\mu = 2$.

Apart from the standard parameters, there are others that are specific to the model, and are also held constant throughout the four equilibria. These are the curvature parameter of the probability of being audited function, γ, the punishment that has to be paid if caught working in the informal sector, ϕ, the returns to scale of the informal production function, ν, the persistence of informal productivity, ρ_z, and the standard deviation of informal productivity, σ_z.

There is no direct evidence on what the curvature parameter in the function that determines the probability of an audit is, so I set $\gamma = 1$ and do a sensitivity analysis for this value. As shown in the results section, the model implications are not affected by changes in this parameter. This is also the case for the punishment term, ϕ. There is no consensus on how big is the penalty that has to be paid if found working on the informal sector. As a result, I fix $\phi = 1$ in the benchmark calibration, and run a sensitivity analysis with respect to this parameter. Table 2 shows the benchmark values as well as the range used in the sensitivity analysis for both γ and ϕ.

As large part of the literature, I assume that the informal sector only uses labor for production and there are decreasing returns to scale on it. The parameter determining the degree of decreasing returns, ν, is key for determining the debt to GDP ratio in the model as well as the distribution of assets. I set ν such that the model generates the qualitative behavior of Net Foreign Assets to GDP

Figure 4.1: Tax Rate vs. Informal Economy



Source: Data from Aruoba (2010).

ratio for the four economies of interest, $\nu = 0.3$.[13]

In general, when we think of formal productivity as being the Solow residual, its persistence is around 0.9 and its standard deviation around 0.007. Even though there is no direct evidence, due to financial constraints and other differences it is natural to think that the unconditional variance of informal productivity should be higher than that of formal productivity. Hence, I set the standard deviation for the informal productivity shock at $\sigma_z = 0.01$ and the autocorrelation coefficient to $\rho_z = 0.8$. In the results section I show a sensitivity analysis with respect to these two parameters.

The remaining two parameters, the tax rate τ and the mean of the informal productivity shock $\bar{z}$ are set to four different values in order to generate the four different stationary equilibria resembling the characteristics of countries such as Austria, Spain, South Africa, and Philippines. Figure 4.1 shows a scatterplot of the tax rate versus the size of the informal economy for most of the countries in Figure 2.1. As one can see tax rates are generally lower for countries with larger informal sectors. For the calibration I take the exact values for the tax rates shown in the figure for Austria, Spain, South Africa, and Philippines.

The average relative informal productivity determines the size of the informal sector. Hence, it varies between equilibria, and is fixed such that the model generates the estimated size of the informal sector in Austria, Spain, South Africa, and Philippines. Table 2 shows the values that the

[13] See the average Net Foreign Assets to GDP ratio in the updated and extended version of the dataset by Lane and Milesi-Ferreti (2007) for the 1983-2001 period for Austria, Spain, South Africa and Philippines. This period corresponds to the period used to calculate the relative volatility of consumption in Neumeyer and Perri (2005).

Table 2: This table shows the range of vales used for the sensitivity analysis for each parameter, as well as its benchmark value (bold letters) and the calibration of $\bar{z}$ and τ in the four equilibriums.

Parameter	**Parameter Value**
$\bar{z}$	Austria=0.3, Spain=0.52, South Africa=0.63, Philippines=0.85
τ	Austria=0.52, Spain=0.39, South Africa=0.33, Philippines=0.2
γ	0.8, **1**, 1.2
ϕ	**1**, 1.1, 1.2
ρ_z	0.75, **0.80**, 0.85
σ_z	0.008, **0.01**, 0.012

parameter takes in each equilibrium. Note that having relative productivity differences between the formal and informal sector can be a simplified way of modeling asymmetric financial constraints. In other words, we know from Quintin (2008) that the degree of contract enforcement in the informal sector plays a crucial role in determining its size. When there is a lack of contract enforcement, there is limited commitment and hence access to credit is restricted. If the informal sector has a restricted access to credit relative to the formal sector, this can be reflected in productivity differentials as the informal sector won't have enough resources to devote to research and development or absorb technological spillovers.

Table 2 shows a summary of the benchmark parameter values (in bold) as well as the ranges used in the sensitivity analysis in Section 5.2.

Finally, I assume that formal and informal consumption goods are perfect substitutes and that the utility function is given by $u(c_t^f, c_t^i) = \frac{\left(c_t^f + c_t^i\right)^{1-\mu} - 1}{1-\mu}$.[14] I consider this to be a natural assumption as evidence indicates that formal and informal consumption goods are very good substitutes. Two of the sectors with the highest share of informal workers are commerce and domestic servants, (Gasparini and Tornarolli (2009)). For example, there is a large degree of informality in the retail sector, and it is generally the case that consumption goods sold by a formal vendor are the same as those sold by an informal one. To illustrate this, there are two very good and different examples that give a sense of the range of goods that can be produced by the two sectors and of their degree of substitutability. First, the fruit and/or vegetable that is sold by a vendor on the street is the same one that is found on the supermarket. Similarly, in the case of electronics, developing countries have malls that sell TV's, DVD players, Playstations etc. at lower prices and it is well known that this is because they do not pay any taxes on them, but it is exactly the same product that one would

[14]This assumption implies that in equilibrium the relative price of informal goods, q, is equal to 1 because the marginal utility of formal and informal goods is the same. However, note that even though $c^i = 0$ cannot be an equilibrium (because whenever $y^i = 0$ the marginal product of n^i is infinite), $c^f = 0$ can be an equilibrium if the international interest rate is sufficiently high. Nevertheless, I will abstract from this equilibrium as we know that there is no country that relies one hundred percent on the informal sector and thus has zero formal consumption.

find at a Sony or Apple Store.[15]

In order to solve for each of the four stationary equilibria, I start by fixing r.[16] I then set initial values for $\{w_i, T_i\}$, compute policy functions $c(z,a)$ and $a'(z,a)$, compute $\{n,a,k,y^f,y^i,c\}$ and update w and T. If $\left\| \begin{array}{c} w_{i+1}-w_i \\ T_{i+1}-T_i \end{array} \right\| < 0$, the equilibrium has been reached; otherwise, I use the updated values w and T, go back and compute policy functions $c(z,a)$ and $a'(z,a)$, and repeat the same procedure.

Once I have the stationary equilibrium for the four economies, I calculate the theoretical moments for each of the four cases and I calculate the relative volatility of consumption to output by calculating the ratio of the standard deviation of the two variables of interest.

5 Results

5.1 Benchmark Calibration

As discussed in Section 2, it is very difficult to know to what extent the informal sector is measured in the National Accounts of a country. Hence, I take the two extreme cases, what if *none* of it is captured and what if *all* of it is captured in the official estimates.

Figure 5.1 shows the results for the benchmark calibration. The scatterplot represents the data points shown in Figure 2.1, the solid line in the graph is the model-generated relative volatility of consumption to output when only formal consumption and output are measured ($\sigma(c^f)/\sigma(y^f)$, *Formal Relative Volatility*) and the dashed line is the trend for the model-generated relative volatility of consumption to output when both formal and informal consumption and output are measured ($\sigma(c^T)/\sigma(y^T)$, *Total Relative Volatility*).

The main mechanism driving the results of the model relies on the fact that when individuals are able to substitute between formal and informal consumption goods over time, volatility arises due to relative productivity differentials between the formal and the informal sectors.

[15] These malls are known by different names across countries (cities). A few examples in Mexico City are "Tepito" and "Plaza Meave," in Perú (Lima) the largest one is called "Polvos Azules," in Agentina (Buenos Aires) "Galería Jardín," in Colombia (Bogotá) "San Andresito," and in Colombia (Medellín) "El Hueco," to name a few.

Usually they also sell CD's, DVD's, and designer clothes. Most of the merchandise is original, but they sell it at lower prices because they don't pay any sales taxes on it. Nevertheless, it comes with a factory warranty. One can tell that a small part of the merchandise is not original but a very close replica. This can be the case of CD's, DVD's or clothes; when they are not the originals they are sold at extremely low prices.

Although the prices at these malls are always lower than in formal stores, there are several reasons why not everyone shops there and would rather go to formal stores. These malls are usually located in unsafe, low-income areas of the city, so people often prefer formal stores in order to avoid the hassle of going there, both because of the commuting and personal safety. At the same time, those that live close by or are not threatened by the area may prefer going to a formal store in order to have access to credit.

[16] Due to the small open economy assumption.

Figure 5.1: Model generated relative volatility of consumption to output vs the Informal Sector (the solid line corresponds to the Formal Relative Volatility and the dashed line corresponds to the Total Relative Volatility).

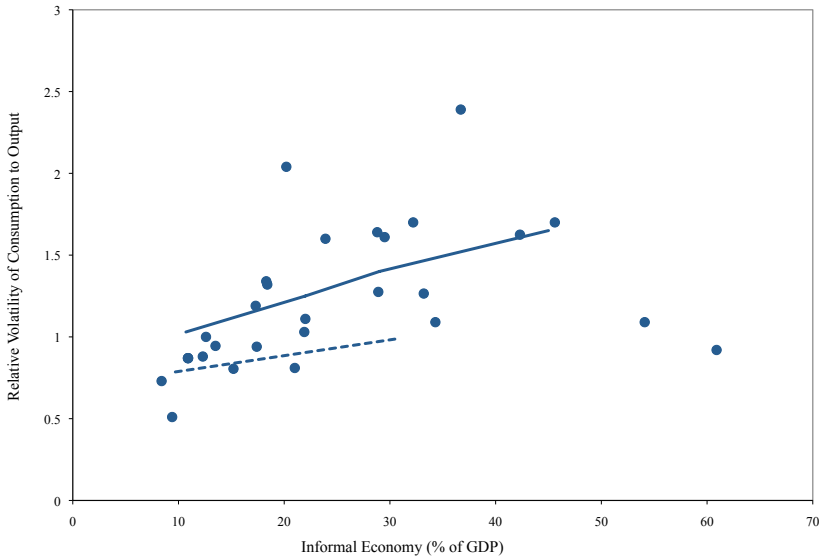
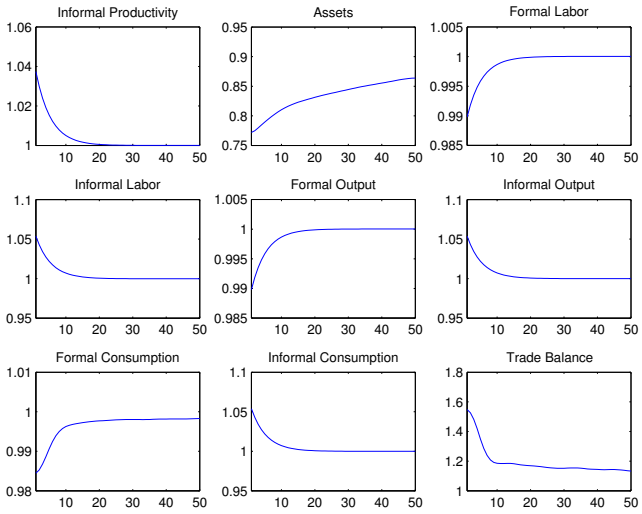


Figure 5.2 shows the response of the main variables to an increase in relative informal productivity, z_t. If informal productivity is suddenly higher, agents move labor from the formal to the informal sector (n^f goes down and n^i increases) and substitute formal for informal consumption (c^f goes down and c^i increases). This reduction in formal labor generates a drop in formal output, y^f, and an increase in informal output, y^i.[17] Note that as only formal goods are traded,[18] all consumption smoothing is done through the formal sector. This means that even when the informal sector becomes more attractive due to a positive relative productivity shock, formal output will not adjust too much because of a strategic savings motive. Agents know that they face a risk of a tax audit when working in the informal sector, and that they can loose a fraction of their production. As a result they will insure themselves against future uncertainty through changes in their net foreign asset position. As only formal goods are traded, formal output is determined by $y_t^f = c_t^f - (1+r_t)a_t + a_{t+1}$, where $TB_t = a_{t+1} - (1+r_t)a_t$ is the trade balance. In the extreme case that *only formal output and formal consumption are measured*, this means that the volatility of output will only be less than that of consumption if the covariance between formal consumption and the trade balance is sufficiently negative. Here, due to the strategic savings motive, when un-

[17]There is empirical evidence that supports the counter-cyclical behavior of the informal sector, (see Fernandez and Meza (2014)).

[18]There is evidence that the sectors with the largest share of informal workers are non-tradable, see Gasparini and Tornarolli (2009).

Figure 5.2: Impulse response functions to a positive informal productivity shock



Percentage changes in response to a one standard deviation increase in informal relative productivity.

certainty increases because informal production increases (increasing the risk of a tax audit), then the agent increases tomorrow's net foreign asset position more than today's, generating a negative covariance between c^f and TB_t that is sufficiently large to make the volatility of c^f higher than that of y^f.

The solid line in Figure 5.1 shows that as the size of the informal sector increases in equilibrium, as in the data, the model-generated Formal Relative Volatility increases as well. In an equilibrium with a larger informal sector, the strategic savings motive mentioned above kicks in with more force, generating a debt to GDP ratio that is smaller than in equilibriums were the informal sector is smaller.[19] Nevertheless, within that equilibrium, if informal labor, n^i, increases even further due to a positive relative productivity shock, z, uncertainty decreases because the probability of a tax audit goes down further, the informal sector acts more as insurance, and the strategic savings motive decreases generating a decrease in the net foreign asset position, a_t of the agent. The probability of a tax audit goes down because a larger informal sector is associated with a lower tax rate (see Figure 4.1). This means lower tax revenues (both because the tax rate is lower and the formal sector is smaller). In the model, lower tax revenues result in a weaker rule of law which is represented by the amount of resources the government spends in monitoring the informal sector ($G_t = \tau w_t n_t + p_t \phi \tau y_t^i$) and hence a lower probability of being audited, $p_t = G_t(n_t^i)^\gamma$, (see Figure 5.4). However, even though today's net foreign assets, a_t, are positively correlated with formal consumption, c^f, tomorrow's net foreign asset position, a_{t+1}, is much less so than today's (due

[19]Within the four equilibria for which the model is calculated, the debt to GDP ratio goes from around 40% of formal GDP when the informal sector is 10% of formal GDP, to 30% of formal GDP when the informal sector is 45% of formal GDP. This behavior is qualitatively the same as in the data.

to tomorrow's higher uncertainty), generating a stronger negative covariance between formal consumption, c^f, and the trade balance, TB, than when the informal sector is smaller. As a result, when only the formal sector is measured, as the size of the informal sector increases in equilibrium so does the *Formal Relative Volatility*.

When both formal and informal consumption and output are measured, what we observe is different. The dashed line in Figure 5.1 shows that the *Total Relative Volatility* of consumption to output is smaller than the *Formal Relative Volatility*, and that it is lower than one. This is because formal and informal consumption goods are negatively correlated, so when total consumption is calculated, the volatility of the two goods cancels out resulting in a much lower total consumption volatility. At the same time, due to the strategic savings motive, the trade balance is not as volatile as formal consumption, so the covariance between formal and informal output is lower than that of consumption. As a result, total consumption is less volatile than total output.

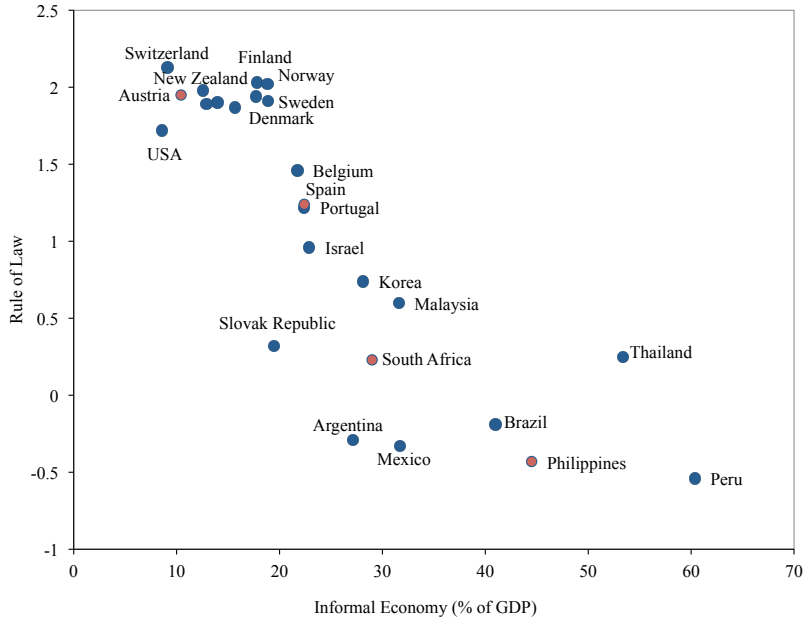
These results show that a country's reported relative volatility of consumption to output can be somewhere between 38% and 77% higher than its true value depending on the size of the informal economy and whether informal output is measured or not. This implies a presumption that the relative volatility of consumption to output will be higher in countries with large informal sectors that are not well measured.

As an outside check, I compare de empirical rule of law (see Figure 5.3) to the model-generated Rule of Law, $G_t = \tau w_t n_t + p_t \phi \tau y_t^i$ (see Figure 5.4, where G_t has been transformed into an index where the maximum value is one). Note that because of the interaction between G_t and p_t, it is not obvious that the model's Rule of Law should be decreasing on the size of the informal sector. It could be that the probability of an audit, p_t, increases with the size of the informal sector (as it is more easily discoverable), in which case G_t could increase, causing a positive relationship between the informal sector and the Rule of Law. On the contrary, if the effect of G_t dominates p_t, as tax revenues decrease, so will the probability of an audit, and G_t will decrease further, creating a negative relationship between the Rule of Law and the informal sector as observed in the data.

5.2 Sensitivity Analysis

Figure 5.5 shows the effect of the persistence of informal relative volatility, ρ_z, on the results. It shows that it has a very small incidence on the Formal Relative Volatility, but that it matters for the Total Relative Volatility. This is because when informal relative volatility, z_t, is less persistent (as illustrated by the green lines of Figure 5.5), overall uncertainty increases, generating a more volatile formal consumption and an even less volatile trade balance because of the increased strategic savings motive. This means that when there is a relative productivity shock, formal consumption moves from one sector to the other in larger quantities than in the benchmark calibration,

Figure 5.3: Rule of Law vs. Informal Economy



Source: Data from Aruoba (2010)

Figure 5.4: Model Generated Rule of Law vs. Informal Economy

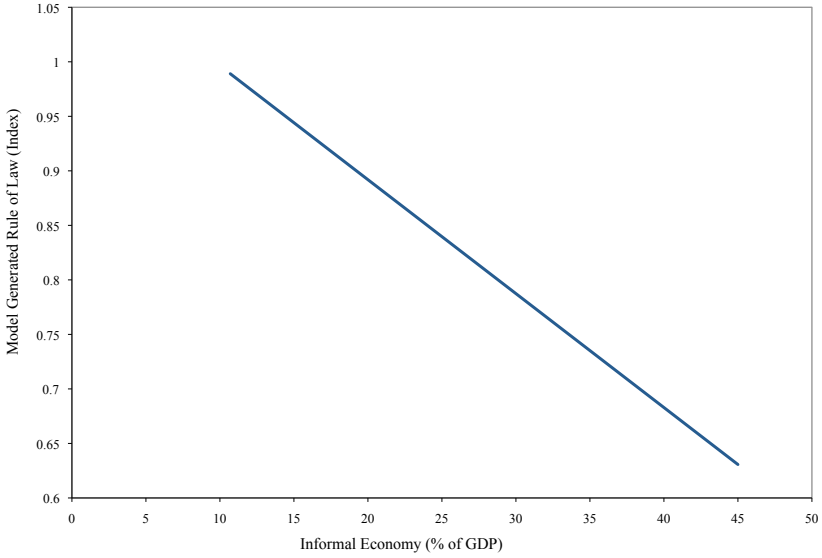
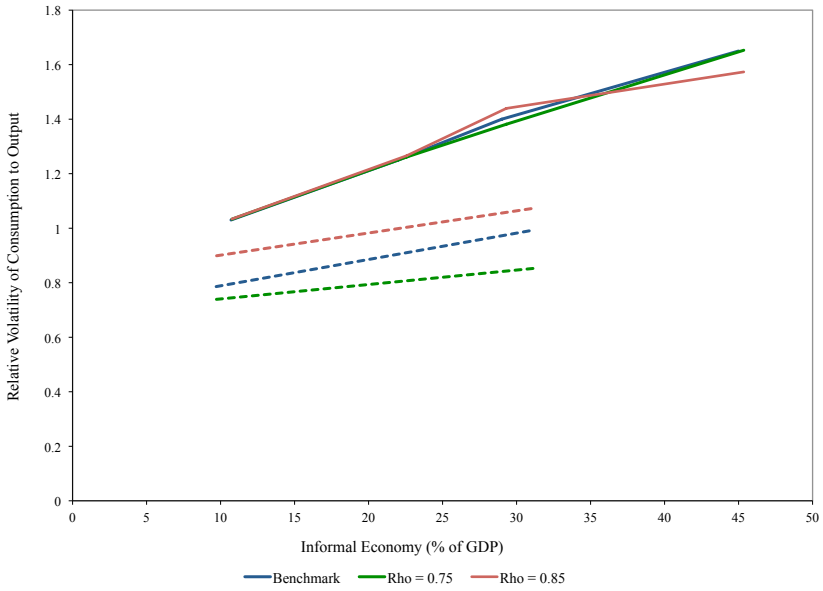


Figure 5.5: Sensitivity Analysis to Persistence of Informal Relative Productivity (the solid lines correspond to the Formal Relative Volatility and the dashed lines correspond to the Total Relative Volatility).



but the trade balance moves less. This implies that when both sectors are measured, the resulting total relative volatility of consumption to output is lower than in the benchmark calibration. The opposite happens when the persistence in higher (see the red dashed line in the figure). As a result, changes in the persistence of informal relative volatility will affect the difference between the "true" relative volatility and the "observed" one, but not the fact that when the informal sector is measured poorly, it is natural to observe a higher relative volatility of consumption to output.

Figure 5.6 shows the sensitivity of the results to the standard deviation of the informal relative productivity, σ_z. We can see that reasonable changes in this parameter do not change the implications of the model.

Figures 5.7 and 5.8 show the sensitivity of the results to the curvature of the probability of being audited function, p_t, and to the penalty if audited, ϕ. Again, they show that the implications of the model are not affected by reasonable changes in these parameters.

6 Final Remarks

This paper has documented a systematic cross-country relationship between the relative volatility of cyclical consumption to output and the size of the informal sector. Specifically, I found that countries that have a higher cyclical volatility of consumption relative to output have large informal sectors.

Figure 5.6: Sensitivity Analysis to Standard Deviation of Informal Productivity (the solid lines correspond to the Formal Relative Volatility and the dashed lines correspond to the Total Relative Volatility).

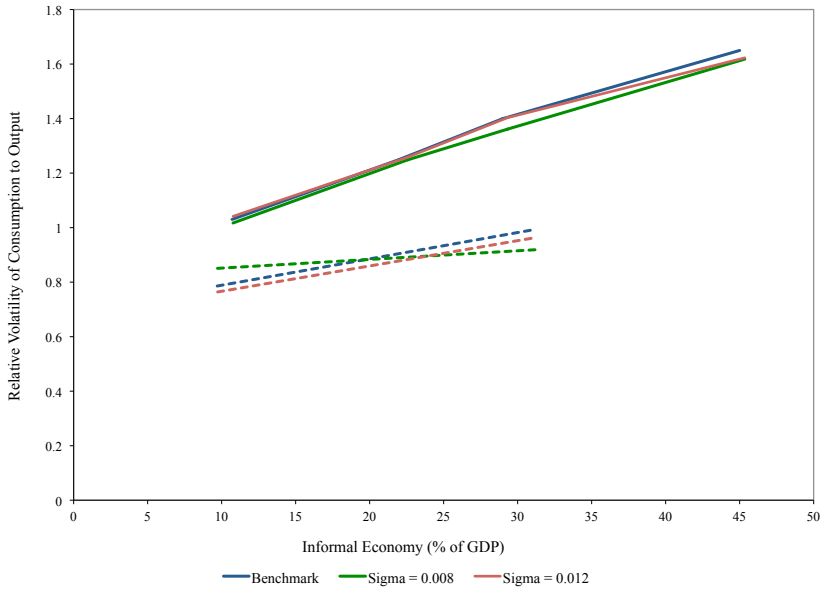


Figure 5.7: Sensitivity Analysis to Curvature of Probability of Being Audited (the solid lines correspond to the Formal Relative Volatility and the dashed lines correspond to the Total Relative Volatility).

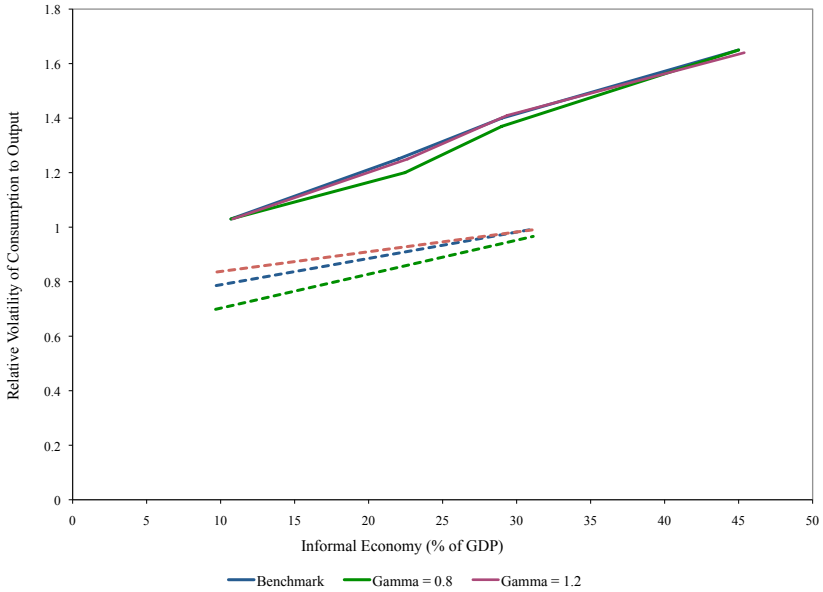
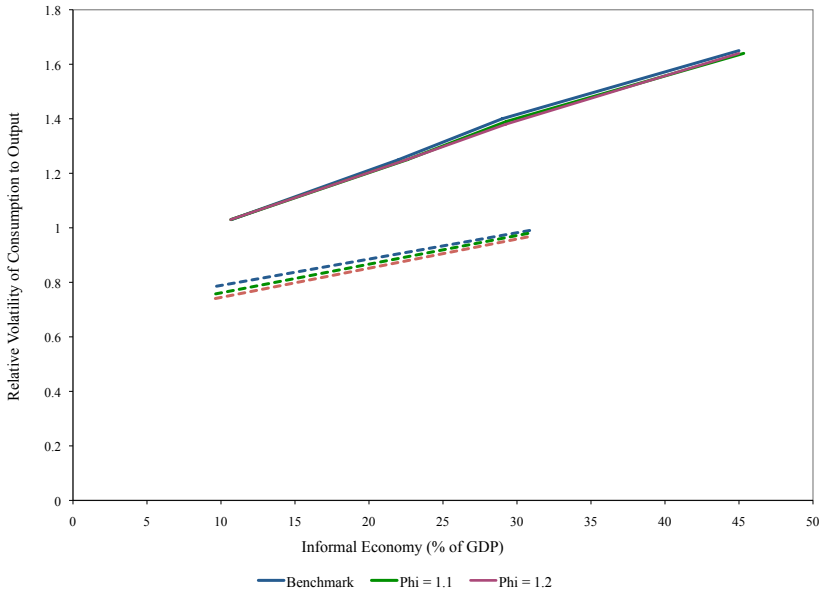


Figure 5.8: Sensitivity Analysis to Penalty if Audited (the solid lines correspond to the Formal Relative Volatility and the dashed lines correspond to the Total Relative Volatility).



To understand this relationship, I constructed a small open economy model in which there are two sectors, a formal sector which is measured and an informal sector which is not measured. The model shows that relatively high consumption volatility is a natural consequence of having a large and poorly measured informal sector. The main factor driving the high volatility is that formal and informal consumption goods are substitutes. The ability of individuals to substitute between formal and informal consumption goods, both over time and within the same period, implies that relative productivity differentials between the formal and the informal sectors generate volatility. Furthermore, I show that when formal goods are tradable, consumption can be more volatile than output. This gives rise to the observation that the relative volatility of consumption to output is greater than one in countries with large unmeasured informal sectors.

This "informal sector" channel is thus a mechanism driving high consumption volatility which is complementary to other mechanisms in the literature, such as country risk that is affected by fundamental shocks and the presence of working capital and shocks to trend growth.

Interestingly, the model can rationalize high observed volatility of measured consumption relative to output in both developing countries and some developed countries that have a large informal economy. My results imply that studies of business cycles in developing countries should recognize the importance of the informal sector and how that mis-measurement can affect measured cyclical fluctuations.

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References

AGUIAR, M. AND G. GOPINATH, "Emerging Market Business Cycles: The Cycle is the Trend," *Journal of Political Economy* 115 (2007), 69–102.

ARUOBA, S. B., "Informal Sector, Government Policy and Institutions," University of Maryland, 2010.

BASSETTO, M. AND C. PHELAN, "Tax Riots," *Review of Economic Studies* 75 (2008), 649–669.

BENHABIB, J., R. ROGERSON AND R. WRIGHT, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99 (1991), 1166–1187.

BOVI, M., "Shadow Employment and Labor Productivity Dynamics," *LABOUR* 21 (December 2007), 735–761.

CAGAN, P., "The Demand for Currency Relative to the Total Money Supply," *Journal of Political Economy* (1958), 302–28.

CLOTFELTER, C. T., "Tax Evasion and Tax Rates: An analysis of Individual Return," *Review of Economics and Statistics* (1983), 363–73.

DALLAGO, B., *The Irregular Economy: The Underground Economy and the Black Labor Market* (Dartmouth: U. K. Publishing Co., 1990).

FEIGE, E. L., "How Big is the Irregular Economy?," *Challenge* (1979), 5–13.

———, "A Re-examination of the 'Underground Economy' in the United States," *IMF Staff Papers* (1986), 768–81.

FERNANDEZ, A. AND F. MEZA, "Informal Employment and Business Cycles in Emerging Economies: The Case of Mexico," Inter-American Development Bank, 2014.

GASPARINI, L. AND L. TORNAROLLI, "Labor Informaility in Latin America and the Caribbean: Patterns and Trends from Household Survey Microdata," *Revista Desarrollo y Sociedad* (2009).

IRS, "Estimates of Income Unreported on Individual Tax Returns," Technical Report, IRS, U.S. Treasury Dept., 1979.

———, "Income Tax Compliance Research: Estimates for 1973-81," Technical Report, IRS, U.S. Treasury Dept., 1983.

KAUFMANN, D. AND A. KALIBERDA, "Integrating the Unofficial Economy into the Dynamics of Post Socialist Economies: A Framework of Analysis and Evidence," Working paper, World Bank Policy Research Working Paper 1691, 1996.

LANE, P. R. AND G. M. MILESI-FERRETI, "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," *Journal of International Economics* 73 (2007), 223–250.

MAGNAC, T., "Segmented or Competitive Labor Markets?," *Econometrica* 59 (1991), 165–187.

MALONEY, W. F., "Does Informality Imply Segmentation in Urban Labor Markets? Evidence from Sectoral Transitions in Mexico," *The World Bank Economic Review* 13 (1999), 275–302.

MENDOZA, E. G., "The Terms of Trade, The Real Interest Rate, and Economic Fluctuations," *International Economic Review* 36 (1995), 101–137.

MOGENSEN, G. V., H. K. KVIST, E. KORMENDI AND S. PEDERSEN, "The Shadow Economy in Denmark 1994: Measurement and Results," Technical Report, Rockwool Foundation Research Unit, Copenhagen, 1995.

NEUMEYER, P. A. AND F. PERRI, "Business Cycles in Emerging Economies: The Role of Interest Rates," *Journal of Monetary Economics* 52 (2005), 345–380.

OECD, *System of National Accounts, 1993 - Glossary* (OECD Publishing, 2000).

———, *Measuring the Non-Observed Economy-A Handbook* (OECD, 2002).

PRATAP, S. AND E. QUINTIN, "Are Labor Markets Segmented in Developing Countries? A Semi-parametric Approach," *European Economic Review* 50 (2006), 1817–1841.

QUINTIN, E., "Contract enforcement and the size of the informal economy," *Economic Theory* 37 (2008), 395–416.

RAND, J. AND F. TARP, "Business Cycles in Developing Countries: Are they Different?," *World Development* 30 (2002), 2071–2088.

SCHNEIDER, F., "Shadow Economies and Corruption all over the World: New Estimates for 145 Countries," *Economics: The Open-Access, Open-Assessment E-Journal* 1 (2007), 1–66.

SCHNEIDER, F. AND D. H. ENSTE, "Shadow Economies: Size, Causes and Consequences," *Journal of Economic Literature* 38 (2000), 77–114.

SIMON, C. P. AND A. D. WITTE, *Beating the System: The Underground Economy* (Boston: Urban House, 1982).

TANZI, V., "The Underground Economy in the United States: Estimates and Implications," Technical Report 135, Banca Nazionale del Lavoro, 1980.

———, "The Underground Economy in the United States: Estimates and Implications," *Staff Papers - International Monetary Fund* 30 (1983), 283–305.

THOMAS, J. J., *Informal Economic Activity* (The University of Michigan Press, 1992).

WITTE, A. D., "The Nature and Extent of Unreported Activity: A Survey Concentrating on Recent US Research," (1987).

Appendix

There are two types of methods which are most commonly used to measure the size of the informal economy. These are the Direct Approach and the Indirect Approach (Schneider and Enste (2000)). Here, I will briefly present what each method consists of and how it can be applied.

Direct Approaches

This is a method that relies on surveys, samples based on voluntary replies, tax auditing and other compliance methods. The problem with this approach is that the results are going to depend directly on the questions asked by the survey. Therefore, it is usually the case that the surveys made in every country (i.e. the Household Surveys) contain different questions, so it is very difficult to try to measure the informal economy of different countries using the same parameters.

Usually what ends up happening is that the definition used has to be very simple and contain only one parameter. For example, the informal sector may be defined as those people who do not have the right for a pension when they are old, but clearly this definition excludes several important elements that would describe the informal economy otherwise. The other very common definition is that a person is considered to work in the informal economy if they work for a firm that has N or fewer workers. This is a very strong assumption because a firm can be very small but still comply with the law and its production can be reported to the authorities, meaning that its value added will appear in the GDP no matter how small it is.

On the other hand, if what is used is a direct questionnaire, people are not usually willing to admit that they are not reporting taxes or that they are having a fraudulent behavior, either because they feel afraid of getting caught or because they feel ashamed since they know this is a moral issue. This makes it difficult to estimate the extent of undeclared work. The advantages and disadvantages of this method are discussed in great detail by Mogensen et al. (1995).

Finally, a direct estimate of the informal economy can also be obtained by calculating the discrepancy between income declared for tax purposes and that measured by selective checks. This method has been used by the IRS (1979) and IRS (1983) in the United States, Simon and Witte (1982), Witte (1987), Clotfelter (1983), and Feige (1986). Good discussions can be found in Dallago (1990) and Thomas (1992).

Indirect Approaches

These are macroeconomic approaches, known as indicator approaches. Here are some of them.

Discrepancy between the National Expenditure and Income Statistics In theory, the income measure of the GDP and the expenditure measure should be equal to each other. However, informal

activities will show up in the expenditure measurement but not in the income measurement. Thus, the difference between these two measures is an indicator of the size of the informal economy. The problem with this estimate is that the statisticians would like to make the difference between the two as small as possible, so it would be ideal to take the initial measure and not the published measure. Then, apart from this issue, we also have the differences due to sampling and statistical error, which can not be disentangled from the amount that can be explained by the informal economy.

Discrepancy between Official and Actual Labor Force Assuming that the total labor force participation is constant, ceteris paribus, then any decrease in the labor force participation in the official economy can be seen as an indicator of an increase in the activity in the informal economy. The problem with this method is that changes in the labor force participation can be due to other causes. It could also be the case that people work in both the informal and formal economy, so this is not a very good estimator.

The Transactions Approach Feige (1979) developed this approach based on the quantitative theory of money $MV = pT$, where M is money, V is velocity, p are prices, and T total transactions. The main assumption is that the relation over time of the volume of transactions and official GNP is constant. He uses the value of total transactions (pT) as an estimate of nominal GNP; then, the informal economy is the difference between nominal GNP and the official GNP. The problem is that he has to assume a base year where the assumption is that there is no informal economy. Also, the assumption that the ratio of transactions to official GNP is constant over time is quite strong. Additionally, obtaining accurate estimates of the total number of transactions is difficult.

The Currency Demand Approach This approach was first proposed by Cagan (1958) and then Tanzi (1980), and Tanzi (1983) took the method a step further. The main idea is that they want to see the correlation between currency demand and tax pressure. The main assumption is that informal activities operate with cash. Thus, if the tax burden increases and so does the demand for money then that increase in the demand for money reflects an increase in the informal economy.

In order to calculate the excess in money demand, they estimate an equation for money demand using econometric methods. They control for development of income, payment habits, interest rates, and other related variables. In the equation they also include government regulation, direct and indirect tax burden, and the complexity of the tax system.

The most common critiques to this approach are the following: Not all the transactions in the shadow economy are paid in cash. Most studies using this approach include only the tax burden factor and ignore others such as "tax morality," regulation, and attitudes towards the state (there is usually no reliable data on these factors). A rise in currency demand deposits is usually due

in large degree to a slowdown in demand deposits and not in a rise in currency due to informal economic activity. Also, most studies assume that both the formal and informal economy have the same velocity of money.

The Physical Input (Electricity Consumption) Method Here I will present the *Kaufmann-Kaliberda Method.* Kaufmann and Kaliberda (1996) assume that electricity consumption is the best physical indicator of both formal and informal economic activity. It has been observed that the electricity/GDP elasticity is usually close to one. So, by using electricity as a proxy for the overall economic activity and then subtracting from it the official estimates of GDP we get an indicator of informal economic activity. The difference between the growth of electricity consumption and official GDP is then attributed to the growth of the informal economy.

The critiques to this approach rely in the fact that not all informal activities require a considerable amount of electricity, or, if they do, other energy sources such as gas, oil, coal etc. could be used. Also, the use of electricity has become more and more efficient in both types of economies. Finally, there may be differences in the elasticity of electricity/GDP across countries or changes over time.